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Title: Mathematica Output for the upper bound when $B=15$, $K=13$ and $\epsilon=1/12$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,a,a,a,a,a,a,a,a,a,a,a\},\{d,d,d,d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335957,\{a>0.0666667,d->0.0666667\}\}$

Point number 2 of type $\{\{0,a,a,a,a,a,a,a,a,a,a,a,a\},\{1-14 d,d,d,d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335304,\{a>0.0714286,d->0.00595238\}\}$

Point number 3 of type $\{\{0,a,a,a,a,a,a,a,a,a,a,a,1-13 a\},\{1-13 d,d,d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335305,\{a>0.00641026,d->0.0714181\}\}$

Point number 4 of type $\{\{0,0,a,a,a,a,a,a,a,a,a,a,a\},\{1/2 (1-13 d),1/2 (1-13 d),d,d,d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.0021801,\{a>0.0769231,d->0.0646887\}\}$

Point number 5 of type $\{\{0,0,a,a,a,a,a,a,a,a,a,a,a,1-12 a\},\{1/2 (1-12 d),1/2 (1-12 d),d,d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335305,\{a>0.00694444,d->0.0714059\}\}$

Point number 6 of type $\{\{0,0,a,a,a,a,a,a,a,a,a,a,1/2 (1-11 a),1/2 (1-11 a)\},\{1/2 (1-11 d),1/2 (1-11 d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.0016148,\{a>0.00757576,d->0.0769038\}\}$

Point number 7 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a,a\},\{1/3 (1-12 d),1/3 (1-12 d),1/3 (1-12 d),d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00187984,\{a>0.0833333,d->0.0645337\}\}$

Point number 8 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a,1-11 a\},\{1/3 (1-11 d),1/3 (1-11 d),1/3 (1-11 d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335306,\{a>0.00757576,d->0.0713914\}\}$

Point number 9 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a,1/2 (1-10 a),1/2 (1-10 a)\},\{1/3 (1-10 d),1/3 (1-10 d),1/3 (1-10 d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.0016148,\{a>0.00833333,d->0.0768913\}\}$

Point number 10 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a,1/3 (1-9 a),1/3 (1-9 a),1/3 (1-9 a)\},\{1/3 (1-9 d),1/3 (1-9 d),1/3 (1-9 d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.000595433,\{a>0.00925926,d->0.0833125\}\}$

Point number 11 of type $\{\{0,0,0,0,a,a,a,a,a,a,a,a,a,a\},\{1/4 (1-11 d),1/4 (1-11 d),1/4 (1-11 d),1/4 (1-11 d),d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00174278,\{a>0.0909091,d->0.0642376\}\}$

Point number 12 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,a,1-10\ a\}, \{1/4\ (1-10\ d), 1/4\ (1-10\ d), 1/4\ (1-10\ d), 1/4\ (1-10\ d), d,d,d,d,d,d,d,d,d,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335306, \{a->0.00833333, d->0.071374\} \}$

Point number 13 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,a,1/2\ (1-9\ a), 1/2\ (1-9\ a)\}, \{1/4\ (1-9\ d), 1/4\ (1-9\ d), 1/4\ (1-9\ d), 1/4\ (1-9\ d), d,d,d,d,d,d,d,d,d,0,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.0016148, \{a->0.00925926, d->0.076876\} \}$

Point number 14 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,a,1/3\ (1-8\ a), 1/3\ (1-8\ a), 1/3\ (1-8\ a)\}, \{1/4\ (1-8\ d), 1/4\ (1-8\ d), 1/4\ (1-8\ d), 1/4\ (1-8\ d), d,d,d,d,d,d,d,d,d,0,0,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.000595433, \{a->0.0104167, d->0.083302\} \}$

Point number 15 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,a,1/4\ (1-7\ a), 1/4\ (1-7\ a), 1/4\ (1-7\ a), 1/4\ (1-7\ a)\}, \{1/4\ (1-7\ d), 1/4\ (1-7\ d), 1/4\ (1-7\ d), 1/4\ (1-7\ d), d,d,d,d,d,d,d,d,d,0,0,0,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.000128248, \{a->0.0119048, d->0.090909\} \}$

Point number 16 of type $\{ \{11/12,a,a,a,a,a,a,a,a,a,a,a,a,a\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d,d\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335304, \{a->0.00595238, d->0.0714286\} \}$

Point number 17 of type $\{ \{11/12,a,a,a,a,a,a,a,a,a,a,a,a,a,1/12-13\ a\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d,d,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00173729, \{a->0., d->0.0769231\} \}$

Point number 18 of type $\{ \{11/12,a,a,a,a,a,a,a,a,a,a,a,a,a,1/2\ (1/12-12\ a), 1/2\ (1/12-12\ a)\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d,d,0,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.000644679, \{a->0., d->0.0833333\} \}$

Point number 19 of type $\{ \{11/12,a,a,a,a,a,a,a,a,a,a,a,a,a,1/3\ (1/12-11\ a), 1/3\ (1/12-11\ a), 1/3\ (1/12-11\ a)\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d,d,0,0,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.000139906, \{a->0., d->0.0909091\} \}$

Point number 20 of type $\{ \{11/12,0,a,a,a,a,a,a,a,a,a,a,a,a,a\}, \{0,1-13\ d,d,d,d,d,d,d,d,d,d,d,d,d,d\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00335305, \{a->0.00641026, d->0.0714181\} \}$

Point number 21 of type $\{ \{11/12,0,a,a,a,a,a,a,a,a,a,a,a,a,a,1/12-12\ a\}, \{0,1-12\ d,d,d,d,d,d,d,d,d,d,d,d,d,d,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.00173729, \{a->0., d->0.0769231\} \}$

Point number 22 of type $\{ \{11/12,0,a,a,a,a,a,a,a,a,a,a,a,a,a,1/2\ (1/12-11\ a), 1/2\ (1/12-11\ a)\}, \{0,1-11\ d,d,d,d,d,d,d,d,d,d,d,d,d,d,0,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.000644679, \{a->0., d->0.0833333\} \}$

Point number 23 of type $\{ \{11/12,0,a,a,a,a,a,a,a,a,a,a,a,a,a,1/3\ (1/12-10\ a), 1/3\ (1/12-10\ a), 1/3\ (1/12-10\ a)\}, \{0,1-10\ d,d,d,d,d,d,d,d,d,d,d,d,d,d,0,0,0\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to $\{0.000139906, \{a->0., d->0.0909091\} \}$

Point number 24 of type $\{ \{11/12,0,0,a,a,a,a,a,a,a,a,a,a,a,a,a\}, \{0,1/2\ (1-12\ d), 1/2\ (1-12\ d), d,d,d,d,d,d,d,d,d,d,d,d,d\} \}$ with $\{a,d\} \leq 1/12$ has maximum value equal to

{0.00335305,{a->0.00694444,d->0.0714059}}

Point number 25 of type {{11/12,0,0,a,a,a,a,a,a,a,a,1/12-11 a},{0,1/2 (1-11 d),1/2 (1-11 d),d,d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/12 has maximum value equal to {0.00173729,{a->0.,d->0.0769231}}

Point number 26 of type {{11/12,0,0,a,a,a,a,a,a,a,a,1/2 (1/12-10 a),1/2 (1/12-10 a)},{0,1/2 (1-10 d),1/2 (1-10 d),d,d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/12 has maximum value equal to {0.000644679,{a->0.,d->0.0833333}}

Point number 27 of type {{11/12,0,0,a,a,a,a,a,a,a,a,1/3 (1/12-9 a),1/3 (1/12-9 a),1/3 (1/12-9 a)},{0,1/2 (1-9 d),1/2 (1-9 d),d,d,d,d,d,d,d,d,d,0,0,0}} with {a,d} <= 1/12 has maximum value equal to {0.000139906,{a->0.,d->0.0909091}}

Point number 28 of type {{11/12,0,0,0,a,a,a,a,a,a,a,a,a},{0,1/3 (1-11 d),1/3 (1-11 d),1/3 (1-11 d),d,d,d,d,d,d,d,d,d,d}} with {a,d} <= 1/12 has maximum value equal to {0.00335306,{a->0.00757576,d->0.0713914}}

Point number 29 of type {{11/12,0,0,0,a,a,a,a,a,a,a,a,a,1/12-10 a},{0,1/3 (1-10 d),1/3 (1-10 d),1/3 (1-10 d),d,d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/12 has maximum value equal to {0.00173729,{a->0.,d->0.0769231}}

Point number 30 of type {{11/12,0,0,0,a,a,a,a,a,a,a,a,a,1/2 (1/12-9 a),1/2 (1/12-9 a)},{0,1/3 (1-9 d),1/3 (1-9 d),1/3 (1-9 d),d,d,d,d,d,d,d,d,d,0,0,0}} with {a,d} <= 1/12 has maximum value equal to {0.000644679,{a->0.,d->0.0833333}}

Point number 31 of type {{11/12,0,0,0,a,a,a,a,a,a,a,a,a,1/3 (1/12-8 a),1/3 (1/12-8 a),1/3 (1/12-8 a)},{0,1/3 (1-8 d),1/3 (1-8 d),1/3 (1-8 d),d,d,d,d,d,d,d,d,d,0,0,0}} with {a,d} <= 1/12 has maximum value equal to {0.000139906,{a->0.,d->0.0909091}}

Point number 32 of type {{11/12,0,0,0,0,a,a,a,a,a,a,a,a,a,a},{0,1/4 (1-10 d),1/4 (1-10 d),1/4 (1-10 d),1/4 (1-10 d),d,d,d,d,d,d,d,d,d,d}} with {a,d} <= 1/12 has maximum value equal to {0.00335306,{a->0.00833333,d->0.071374}}

Point number 33 of type {{11/12,0,0,0,0,a,a,a,a,a,a,a,a,a,1/12-9 a},{0,1/4 (1-9 d),1/4 (1-9 d),1/4 (1-9 d),1/4 (1-9 d),d,d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/12 has maximum value equal to {0.00173729,{a->0.,d->0.0769231}}

Point number 34 of type {{11/12,0,0,0,0,a,a,a,a,a,a,a,a,a,1/2 (1/12-8 a),1/2 (1/12-8 a)},{0,1/4 (1-8 d),1/4 (1-8 d),1/4 (1-8 d),1/4 (1-8 d),d,d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/12 has maximum value equal to {0.000644679,{a->0.,d->0.0833333}}

Point number 35 of type {{11/12,0,0,0,0,a,a,a,a,a,a,a,a,a,1/3 (1/12-7 a),1/3 (1/12-7 a),1/3 (1/12-7 a)},{0,1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),d,d,d,d,d,d,d,d,d,0,0,0}} with {a,d} <= 1/12 has maximum value equal to {0.000139906,{a->0.,d->0.0909091}}

Point number 36 of type {{11/12,a,a,a,a,a,a,a,a,a,a,a,a,0},{0,d,d,d,d,d,d,d,d,d,d,d,d,11/12}} with {a,d} <= 1/12 has maximum value equal to {2.29915*10^-12,{a->0.00641026,d->0.00641026}}

Point number 37 of type $\{\{11/12, 0, a, a, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/12-12 d, d, d, d, d, d, d, d, d, d, d, d, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{1.73994 \cdot 10^{-12}, \{a > 0.00694444, d > 0.00640653\}\}$

Point number 38 of type $\{\{11/12, 0, a, a, a, a, a, a, a, a, a, 1/12-11 a, 0\}, \{0, 1/12-11 d, d, d, d, d, d, d, d, d, d, d, 0, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{1.1813 \cdot 10^{-12}, \{a > 0.00693822, d > 0.00693822\}\}$

Point number 39 of type $\{\{11/12, 0, 0, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/12-11 d), 1/2 (1/12-11 d), d, d, d, d, d, d, d, d, d, d, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{1.38432 \cdot 10^{-12}, \{a > 0.00757576, d > 0.00640543\}\}$

Point number 40 of type $\{\{11/12, 0, 0, a, a, a, a, a, a, a, a, a, 1/12-10 a, 0\}, \{0, 1/2 (1/12-10 d), 1/2 (1/12-10 d), d, d, d, d, d, d, d, d, d, 0, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{8.25989 \cdot 10^{-13}, \{a > 0.00756071, d > 0.00693782\}\}$

Point number 41 of type $\{\{11/12, 0, 0, a, a, a, a, a, a, a, a, a, 1/2 (1/12-9 a), 1/2 (1/12-9 a), 0\}, \{0, 1/2 (1/12-9 d), 1/2 (1/12-9 d), d, d, d, d, d, d, d, d, d, 0, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{4.70558 \cdot 10^{-13}, \{a > 0.00756211, d > 0.00756211\}\}$

Point number 42 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/3 (1/12-10 d), 1/3 (1/12-10 d), 1/3 (1/12-10 d), d, d, d, d, d, d, d, d, d, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{1.20489 \cdot 10^{-12}, \{a > 0.00833333, d > 0.00640503\}\}$

Point number 43 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, a, a, 1/12-9 a, 0\}, \{0, 1/3 (1/12-9 d), 1/3 (1/12-9 d), 1/3 (1/12-9 d), d, d, d, d, d, d, d, d, d, 0, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{6.46928 \cdot 10^{-13}, \{a > 0.00826281, d > 0.00693929\}\}$

Point number 44 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, a, a, 1/2 (1/12-8 a), 1/2 (1/12-8 a), 0\}, \{0, 1/3 (1/12-8 d), 1/3 (1/12-8 d), 1/3 (1/12-8 d), d, d, d, d, d, d, d, d, d, 0, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{2.91178 \cdot 10^{-13}, \{a > 0.00827505, d > 0.00756894\}\}$

Point number 45 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, a, a, 1/3 (1/12-7 a), 1/3 (1/12-7 a), 1/3 (1/12-7 a), 0\}, \{0, 1/3 (1/12-7 d), 1/3 (1/12-7 d), 1/3 (1/12-7 d), d, d, d, d, d, d, d, 0, 0, 0, 11/12\}\}$ with $\{a, d\} \leq 1/12$ has maximum value equal to $\{1.11306 \cdot 10^{-13}, \{a > 0.00831113, d > 0.00831116\}\}$

Point number 46 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, a, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13, (1-a)/13\}\}$ with $\{a\} \leq 1/12$ has maximum value equal to $\{0.00335373, \{a > 0.0696161\}\}$

Point number 47 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13, 1/13\}\}$ with $\{ \} \leq 1/12$ has maximum value equal to $0.0017372945024114398`$

Point number 48 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12\}\}$ with $\{ \} \leq 1/12$ has maximum value equal to $0.0006446786051097393`$

Point number 49 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11\}\}$ with $\{\} \leq 1/12$ has maximum value equal to 0.00013990594886818849`

Max attained in point 1 with value 0.0033595665441243715`.

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a, a, a, a, a, a, a, a, a, a, a, a, a, a\}, \{d, d, d, d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00335957, \{a \rightarrow 0.0666667, d \rightarrow 0.0666667\}\}$

Point number 2 of type $\{\{0, a, a, a, a, a, a, a, a, a, a, a, a, a\}, \{1-14 d, d, d, d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00358797, \{a \rightarrow 0.0714286, d \rightarrow 0.\}\}$

Point number 3 of type $\{\{0, a, a, a, a, a, a, a, a, a, a, a, 1-13 a\}, \{1-13 d, d, d, d, d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00358797, \{a \rightarrow 0., d \rightarrow 0.0714286\}\}$

Point number 4 of type $\{\{0, 0, a, a, a, a, a, a, a, a, a, a, a, a\}, \{1/2 (1-13 d), 1/2 (1-13 d), d, d, d, d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.0021801, \{a \rightarrow 0.0769231, d \rightarrow 0.0646887\}\}$

Point number 5 of type $\{\{0, 0, a, a, a, a, a, a, a, a, a, a, 1-12 a\}, \{1/2 (1-12 d), 1/2 (1-12 d), d, d, d, d, d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00358797, \{a \rightarrow 0., d \rightarrow 0.0714286\}\}$

Point number 6 of type $\{\{0, 0, a, a, a, a, a, a, a, a, a, 1/2 (1-11 a), 1/2 (1-11 a)\}, \{1/2 (1-11 d), 1/2 (1-11 d), d, d, d, d, d, d, d, d, d, d, d, 0, 0\}\}$ with value $\{0.00173729, \{a \rightarrow 0., d \rightarrow 0.0769231\}\}$

Point number 7 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, a, a, a, a\}, \{1/3 (1-12 d), 1/3 (1-12 d), 1/3 (1-12 d), d, d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00187984, \{a \rightarrow 0.0833333, d \rightarrow 0.0645337\}\}$

Point number 8 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, a, a, 1-11 a\}, \{1/3 (1-11 d), 1/3 (1-11 d), 1/3 (1-11 d), d, d, d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00358797, \{a \rightarrow 0., d \rightarrow 0.0714286\}\}$

Point number 9 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, a, 1/2 (1-10 a), 1/2 (1-10 a)\}, \{1/3 (1-10 d), 1/3 (1-10 d), 1/3 (1-10 d), d, d, d, d, d, d, d, d, d, d, 0, 0\}\}$ with value $\{0.00173729, \{a \rightarrow 0., d \rightarrow 0.0769231\}\}$

Point number 10 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, 1/3 (1-9 a), 1/3 (1-9 a), 1/3 (1-9 a)\}, \{1/3 (1-9 d), 1/3 (1-9 d), 1/3 (1-9 d), d, d, d, d, d, d, d, d, d, 0, 0, 0\}\}$ with value $\{0.000644679, \{a \rightarrow 0., d \rightarrow 0.0833333\}\}$

Point number 11 of type $\{\{0, 0, 0, 0, a, a, a, a, a, a, a, a, a, a\}, \{1/4 (1-11 d), 1/4 (1-11 d), 1/4 (1-11 d), 1/4 (1-11 d), d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00174278, \{a \rightarrow 0.0909091, d \rightarrow 0.0642376\}\}$

Point number 12 of type $\{\{0, 0, 0, 0, a, a, a, a, a, a, a, a, 1-10 a\}, \{1/4 (1-10 d), 1/4 (1-10 d), 1/4 (1-10 d), 1/4 (1-10 d), d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00358797, \{a \rightarrow 0., d \rightarrow 0.\}\}$

>0.0714286}}

Point number 13 of type {{0,0,0,0,a,a,a,a,a,a,a,1/2 (1-9 a),1/2 (1-9 a)},1/4 (1-9 d),1/4 (1-9 d),1/4 (1-9 d),1/4 (1-9 d),d,d,d,d,d,d,d,d,0,0}} with value {0.00173729,{a->0.,d->0.0769231}}

Point number 14 of type {{0,0,0,0,a,a,a,a,a,a,a,1/3 (1-8 a),1/3 (1-8 a),1/3 (1-8 a)},1/4 (1-8 d),1/4 (1-8 d),1/4 (1-8 d),1/4 (1-8 d),d,d,d,d,d,d,d,d,0,0,0}} with value {0.000644679,{a->0.,d->0.0833333}}

Point number 15 of type {{0,0,0,0,a,a,a,a,a,a,a,1/4 (1-7 a),1/4 (1-7 a),1/4 (1-7 a),1/4 (1-7 a)},1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),d,d,d,d,d,d,d,d,0,0,0,0}} with value {0.000139906,{a->0.,d->0.0909091}}

Max attained in point 2 with value 0.003587969224163778`.

EXACT VALUE OF M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type {{11/12,a,a,a,a,a,a,a,a,a,a,a,a},11/12,d,d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {1.0641*10⁻¹³,{a->0.00595238,d->0.00595238}}

Point number 2 of type {{11/12,0,a,a,a,a,a,a,a,a,a,a,a},11/12,1/12-13 d,d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {1.04504*10⁻¹³,{a->0.00641026,d->0.}}

Point number 3 of type {{11/12,0,a,a,a,a,a,a,a,a,a,a,a,1/12-12 a},11/12,1/12-12 d,d,d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {1.04504*10⁻¹³,{a->0.,d->0.00641026}}

Point number 4 of type {{11/12,0,0,a,a,a,a,a,a,a,a,a,a,a},11/12,1/2 (1/12-12 d),1/2 (1/12-12 d),d,d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {7.02003*10⁻¹⁴,{a->0.00694444,d->0.0057571}}

Point number 5 of type {{11/12,0,0,a,a,a,a,a,a,a,a,a,a,a,1/12-11 a},11/12,1/2 (1/12-11 d),1/2 (1/12-11 d),d,d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {1.04504*10⁻¹³,{a->0.,d->0.00641026}}

Point number 6 of type {{11/12,0,0,a,a,a,a,a,a,a,a,a,a,a,1/2 (1/12-10 a),1/2 (1/12-10 a)},11/12,1/2 (1/12-10 d),1/2 (1/12-10 d),d,d,d,d,d,d,d,d,d,d,0,0}} has maximum value equal to {5.33611*10⁻¹⁴,{a->0.,d->0.00694444}}

Point number 7 of type {{11/12,0,0,0,a,a,a,a,a,a,a,a,a,a,a},11/12,1/3 (1/12-11 d),1/3 (1/12-11 d),1/3 (1/12-11 d),d,d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {6.02772*10⁻¹⁴,{a->0.00757576,d->0.00573762}}

Point number 8 of type {{11/12,0,0,0,a,a,a,a,a,a,a,a,a,a,a,1/12-10 a},11/12,1/3 (1/12-10 d),1/3 (1/12-10 d),1/3 (1/12-10 d),d,d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {1.04504*10⁻¹³,{a->0.,d->0.00641026}}

Point number 9 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, 1/2 (1/12-9 a), 1/2 (1/12-9 a)\}, \{11/12, 1/3 (1/12-9 d), 1/3 (1/12-9 d), 1/3 (1/12-9 d), d, d, d, d, d, d, d, 0, 0\}\}$ has maximum value equal to $\{5.33611 \cdot 10^{-14}, \{a > 0., d > 0.00694444\}\}$

Point number 10 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, 1/3 (1/12-8 a), 1/3 (1/12-8 a), 1/3 (1/12-8 a)\}, \{11/12, 1/3 (1/12-8 d), 1/3 (1/12-8 d), 1/3 (1/12-8 d), d, d, d, d, d, d, d, 0, 0\}\}$ has maximum value equal to $\{2.10735 \cdot 10^{-14}, \{a > 0., d > 0.00757576\}\}$

Point number 11 of type $\{\{11/12, 0, 0, 0, 0, a, a, a, a, a, a, a, a\}, \{11/12, 1/4 (1/12-10 d), 1/4 (1/12-10 d), 1/4 (1/12-10 d), d, d, d, d, d, d, d, d\}\}$ has maximum value equal to $\{5.5444 \cdot 10^{-14}, \{a > 0.00833333, d > 0.00570915\}\}$

Point number 12 of type $\{\{11/12, 0, 0, 0, 0, a, a, a, a, a, a, a, 1/12-9 a\}, \{11/12, 1/4 (1/12-9 d), 1/4 (1/12-9 d), 1/4 (1/12-9 d), 1/4 (1/12-9 d), d, d, d, d, d, d, d, 0\}\}$ has maximum value equal to $\{1.04504 \cdot 10^{-13}, \{a > 0., d > 0.00641026\}\}$

Point number 13 of type $\{\{11/12, 0, 0, 0, 0, a, a, a, a, a, a, a, 1/2 (1/12-8 a), 1/2 (1/12-8 a)\}, \{11/12, 1/4 (1/12-8 d), 1/4 (1/12-8 d), 1/4 (1/12-8 d), 1/4 (1/12-8 d), d, d, d, d, d, d, d, 0, 0\}\}$ has maximum value equal to $\{5.33611 \cdot 10^{-14}, \{a > 0., d > 0.00694444\}\}$

Point number 14 of type $\{\{11/12, 0, 0, 0, 0, a, a, a, a, a, a, a, 1/3 (1/12-7 a), 1/3 (1/12-7 a), 1/3 (1/12-7 a)\}, \{11/12, 1/4 (1/12-7 d), 1/4 (1/12-7 d), 1/4 (1/12-7 d), 1/4 (1/12-7 d), d, d, d, d, d, d, d, 0, 0, 0\}\}$ has maximum value equal to $\{2.10735 \cdot 10^{-14}, \{a > 0., d > 0.00757576\}\}$

Point number 15 of type $\{\{11/12, 0, 0, 0, 0, a, a, a, a, a, 1/4 (1/12-6 a), 1/4 (1/12-6 a), 1/4 (1/12-6 a), 1/4 (1/12-6 a)\}, \{11/12, 1/4 (1/12-6 d), 1/4 (1/12-6 d), 1/4 (1/12-6 d), 1/4 (1/12-6 d), d, d, d, d, d, 0, 0, 0\}\}$ has maximum value equal to $\{4.92463 \cdot 10^{-15}, \{a > 0., d > 0.00833333\}\}$

Point number 16 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{11/12, a, 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a), 1/13 (1/12-a)\}\}$ has maximum value equal to $\{1.04504 \cdot 10^{-13}, \{a > 0.\}\}$

Point number 17 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{11/12, 0, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156\}\}$ has maximum value equal to $1.0450408862792224 \cdot 10^{-13}$

Point number 18 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{11/12, 0, 0, 1/144, 1/144, 1/144, 1/144, 1/144, 1/144, 1/144, 1/144, 1/144, 1/144, 1/144, 1/144\}\}$ has maximum value equal to $5.336108687111633 \cdot 10^{-14}$

Point number 19 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{11/12, 0, 0, 0, 1/132, 1/132, 1/132, 1/132, 1/132, 1/132, 1/132, 1/132, 1/132, 1/132\}\}$ has maximum value equal to $2.107352765038996 \cdot 10^{-14}$

Point number 20 of type $\{\{11/12, 1/12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{11/12, 0, 0, 0, 0, 1/120, 1/120, 1/120, 1/120, 1/120, 1/120, 1/120, 1/120, 1/120\}\}$ has

maximum value equal to $4.924628233477176 \times 10^{-15}$

Max attained in point 1 with value $1.0640992458321692 \times 10^{-13}$.

EXACT VALUE OF M_4 (Unbalanced-Unbalanced distributions on different coordinates)

Point number 1 of type $\{\{11/12, a, a, a, a, a, a, a, a, a, a, a, 0\}, \{0, d, d, d, d, d, d, d, d, d, d, d, 11/12\}\}$ has maximum value equal to $\{2.29915 \times 10^{-12}, \{a > 0.00641026, d > 0.00641026\}\}$

Point number 2 of type $\{\{11/12, 0, a, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/12-12 d, d, d, d, d, d, d, d, d, d, d, 11/12\}\}$ has maximum value equal to $\{1.73994 \times 10^{-12}, \{a > 0.00694444, d > 0.00640653\}\}$

Point number 3 of type $\{\{11/12, 0, a, a, a, a, a, a, a, a, a, 1/12-11 a, 0\}, \{0, 1/12-11 d, d, d, d, d, d, d, d, d, d, 0, 11/12\}\}$ has maximum value equal to $\{1.1813 \times 10^{-12}, \{a > 0.00693822, d > 0.00693822\}\}$

Point number 4 of type $\{\{11/12, 0, 0, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/12-11 d), 1/2 (1/12-11 d), d, d, d, d, d, d, d, d, d, 11/12\}\}$ has maximum value equal to $\{1.38432 \times 10^{-12}, \{a > 0.00757576, d > 0.00640543\}\}$

Point number 5 of type $\{\{11/12, 0, 0, a, a, a, a, a, a, a, a, a, 1/12-10 a, 0\}, \{0, 1/2 (1/12-10 d), 1/2 (1/12-10 d), d, d, d, d, d, d, d, d, d, 0, 11/12\}\}$ has maximum value equal to $\{8.25989 \times 10^{-13}, \{a > 0.00756071, d > 0.00693782\}\}$

Point number 6 of type $\{\{11/12, 0, 0, a, a, a, a, a, a, a, a, 1/2 (1/12-9 a), 1/2 (1/12-9 a), 0\}, \{0, 1/2 (1/12-9 d), 1/2 (1/12-9 d), d, d, d, d, d, d, d, d, 0, 0, 11/12\}\}$ has maximum value equal to $\{4.70558 \times 10^{-13}, \{a > 0.00756211, d > 0.00756211\}\}$

Point number 7 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/3 (1/12-10 d), 1/3 (1/12-10 d), 1/3 (1/12-10 d), d, d, d, d, d, d, d, d, d, 11/12\}\}$ has maximum value equal to $\{1.20489 \times 10^{-12}, \{a > 0.00833333, d > 0.00640503\}\}$

Point number 8 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, a, 1/12-9 a, 0\}, \{0, 1/3 (1/12-9 d), 1/3 (1/12-9 d), 1/3 (1/12-9 d), d, d, d, d, d, d, d, d, 0, 11/12\}\}$ has maximum value equal to $\{6.46928 \times 10^{-13}, \{a > 0.00826281, d > 0.00693929\}\}$

Point number 9 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, 1/2 (1/12-8 a), 1/2 (1/12-8 a), 0\}, \{0, 1/3 (1/12-8 d), 1/3 (1/12-8 d), 1/3 (1/12-8 d), d, d, d, d, d, d, d, d, 0, 0, 11/12\}\}$ has maximum value equal to $\{2.91178 \times 10^{-13}, \{a > 0.00827505, d > 0.00756894\}\}$

Point number 10 of type $\{\{11/12, 0, 0, 0, a, a, a, a, a, a, a, 1/3 (1/12-7 a), 1/3 (1/12-7 a), 1/3 (1/12-7 a), 0\}, \{0, 1/3 (1/12-7 d), 1/3 (1/12-7 d), 1/3 (1/12-7 d), d, d, d, d, d, d, d, 0, 0, 0, 11/12\}\}$ has maximum value equal to $\{1.11306 \times 10^{-13}, \{a > 0.00831113, d > 0.00831116\}\}$

Point number 11 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 11/12\}\}$ has maximum value equal to $1.2283289770859325 \times 10^{-12}$

Point number 12 of type $\{1,0,0,0,0,0,0,0,0,0,0,0,0,0\}$,
 $\{0,0,1/144,1/144,1/144,1/144,1/144,1/144,1/144,1/144,1/144,1/144,1/144,11/12\}$ has maximum value equal to $6.307887830130337 \cdot 10^{-13}$

Point number 13 of type $\{1,0,0,0,0,0,0,0,0,0,0,0,0,0\}$,
 $\{0,0,0,1/132,1/132,1/132,1/132,1/132,1/132,1/132,1/132,1/132,1/132,11/12\}$ has maximum value equal to $2.5081106922102244 \cdot 10^{-13}$

Point number 14 of type $\{1,0,0,0,0,0,0,0,0,0,0,0,0,0\}$,
 $\{0,0,0,0,1/120,1/120,1/120,1/120,1/120,1/120,1/120,1/120,1/120,1/120,11/12\}$ has maximum value equal to $5.909553880172611 \cdot 10^{-14}$

Max attained in point 1 with value $2.299149903395756 \cdot 10^{-12}$.

UPPER BOUND ON THE RATE

The bound on the rate for $b=15$, $k=13$ and $\epsilon=1/12$ is 0.0033538954912978487 and $\eta_0=0.9401518809224649$