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Title: Mathematica Output for the upper bound when $B=K=8$ and $\epsilon=3/25$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{a,a,a,a,a,a,a\},\{d,d,d,d,d,d,d\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0384521,\{a>0.125,d>0.125\}\}$

Point number 2 of type $\{0,a,a,a,a,a,a\},\{1-7\,d,d,d,d,d,d,d\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0384386,\{a>0.142857,d>0.0171429\}\}$

Point number 3 of type $\{0,a,a,a,a,a,1-6\,a\},\{1-6\,d,d,d,d,d,d,0\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0384367,\{a>0.02,d>0.142765\}\}$

Point number 4 of type $\{0,0,a,a,a,a,a\},\{1/2\,(1-6\,d),1/2\,(1-6\,d),d,d,d,d,d\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0244598,\{a>0.166667,d>0.112249\}\}$

Point number 5 of type $\{0,0,a,a,a,a,1-5\,a\},\{1/2\,(1-5\,d),1/2\,(1-5\,d),d,d,d,d,0\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.038435,\{a>0.024,d>0.142638\}\}$

Point number 6 of type $\{0,0,a,a,a,1/2\,(1-4\,a),1/2\,(1-4\,a)\},\{1/2\,(1-4\,d),1/2\,(1-4\,d),d,d,d,d,0,0\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0136179,\{a>0.03,d>0.16659\}\}$

Point number 7 of type $\{22/25,a,a,a,a,a,a\},\{0,d,d,d,d,d,d\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0384386,\{a>0.0171429,d>0.142857\}\}$

Point number 8 of type $\{22/25,a,a,a,a,a,3/25-6\,a\},\{0,d,d,d,d,d,0\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0154321,\{a>0.,d>0.166667\}\}$

Point number 9 of type $\{22/25,0,a,a,a,a,a\},\{0,1-6\,d,d,d,d,d,d\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0384367,\{a>0.02,d>0.142765\}\}$

Point number 10 of type $\{22/25,0,a,a,a,a,3/25-5\,a\},\{0,1-5\,d,d,d,d,d,0\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0154321,\{a>0.,d>0.166667\}\}$

Point number 11 of type $\{22/25,0,0,a,a,a,a\},\{0,1/2\,(1-5\,d),1/2\,(1-5\,d),d,d,d,d\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.038435,\{a>0.024,d>0.142638\}\}$

Point number 12 of type $\{22/25,0,0,a,a,a,3/25-4\,a\},\{0,1/2\,(1-4\,d),1/2\,(1-4\,d),d,d,d,d,0\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0154321,\{a>0.,d>0.166667\}\}$

Point number 13 of type $\{22/25,a,a,a,a,a,0\},\{0,d,d,d,d,d,22/25\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.0000219783,\{a>0.02,d>0.02\}\}$

Point number 14 of type $\{22/25,0,a,a,a,a,0\},\{0,3/25-5\,d,d,d,d,d,22/25\}$ with $\{a,d\} \leq 3/25$ has maximum value equal to $\{0.000015531,\{a>0.024,d>0.019932\}\}$

Point number 15 of type $\{\{22/25, 0, a, a, a, a, 3/25-4 a, 0\}, \{0, 3/25-4 d, d, d, d, 0, 22/25\}\}$ with $\{a, d\} \leq 3/25$ has maximum value equal to $\{9.12417 \cdot 10^{-6}, \{a \rightarrow 0.0238706, d \rightarrow 0.0238706\}\}$

Point number 16 of type $\{\{22/25, 3/25, 0, 0, 0, 0, 0, 0\}, \{0, a, (1-a)/6, (1-a)/6, (1-a)/6, (1-a)/6, (1-a)/6, (1-a)/6\}\}$ with $\{a\} \leq 3/25$ has maximum value equal to $\{0.0384416, \{a \rightarrow 0.139505\}\}$

Point number 17 of type $\{\{22/25, 3/25, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/6, 1/6, 1/6, 1/6, 1/6, 1/6\}\}$ with $\{\} \leq 3/25$ has maximum value equal to 0.015432098765432098

Max attained in point 1 with value 0.0384521484375 .

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a, a, a, a, a, a, a, a\}, \{d, d, d, d, d, d, d, d\}\}$ with value $\{0.0384521, \{a \rightarrow 0.125, d \rightarrow 0.125\}\}$

Point number 2 of type $\{\{0, a, a, a, a, a, a, a\}, \{1-7 d, d, d, d, d, d, d, d\}\}$ with value $\{0.0428393, \{a \rightarrow 0.142857, d \rightarrow 0.\}\}$

Point number 3 of type $\{\{0, a, a, a, a, a, a, 1-6 a\}, \{1-6 d, d, d, d, d, d, d, 0\}\}$ with value $\{0.0428393, \{a \rightarrow 0., d \rightarrow 0.142857\}\}$

Point number 4 of type $\{\{0, 0, a, a, a, a, a, a\}, \{1/2 (1-6 d), 1/2 (1-6 d), d, d, d, d, d, d\}\}$ with value $\{0.0244598, \{a \rightarrow 0.166667, d \rightarrow 0.112249\}\}$

Point number 5 of type $\{\{0, 0, a, a, a, a, a, 1-5 a\}, \{1/2 (1-5 d), 1/2 (1-5 d), d, d, d, d, d, 0\}\}$ with value $\{0.0428393, \{a \rightarrow 0., d \rightarrow 0.142857\}\}$

Point number 6 of type $\{\{0, 0, a, a, a, a, 1/2 (1-4 a), 1/2 (1-4 a)\}, \{1/2 (1-4 d), 1/2 (1-4 d), d, d, d, d, 0, 0\}\}$ with value $\{0.0154321, \{a \rightarrow 0., d \rightarrow 0.166667\}\}$

Max attained in point 2 with value 0.042839293151662995 .

EXACT VALUE OF M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type $\{\{22/25, a, a, a, a, a, a, a\}, \{22/25, d, d, d, d, d, d, d\}\}$ has maximum value equal to $\{1.58033 \cdot 10^{-6}, \{a \rightarrow 0.0171429, d \rightarrow 0.0171429\}\}$

Point number 2 of type $\{\{22/25, 0, a, a, a, a, a, a\}, \{22/25, 3/25-6 d, d, d, d, d, d, d\}\}$ has maximum value equal to $\{1.50589 \cdot 10^{-6}, \{a \rightarrow 0.02, d \rightarrow 0.\}\}$

Point number 3 of type $\{\{22/25, 0, a, a, a, a, a, 3/25-5 a\}, \{22/25, 3/25-5 d, d, d, d, d, d, 0\}\}$ has maximum value equal to $\{1.50589 \cdot 10^{-6}, \{a \rightarrow 0., d \rightarrow 0.02\}\}$

Point number 4 of type $\{\{22/25, 0, 0, a, a, a, a, a\}, \{22/25, 1/2 (3/25-5 d), 1/2 (3/25-5 d), d, d, d, d, d, d\}\}$ has maximum value equal to $\{1.02497 \cdot 10^{-6}, \{a \rightarrow 0.024, d \rightarrow 0.0149442\}\}$

Point number 5 of type $\{\{22/25, 0, 0, a, a, a, 3/25-4 a\}, \{22/25, 1/2 (3/25-4 d), 1/2 (3/25-4 d), d, d, d, d, 0\}\}$ has maximum value equal to $\{1.50589 \cdot 10^{-6}, \{a > 0., d > 0.02\}\}$

Point number 6 of type $\{\{22/25, 0, 0, a, a, a, 1/2 (3/25-3 a), 1/2 (3/25-3 a)\}, \{22/25, 1/2 (3/25-3 d), 1/2 (3/25-3 d), d, d, d, 0, 0\}\}$ has maximum value equal to $\{6.05414 \cdot 10^{-7}, \{a > 0., d > 0.024\}\}$

Point number 7 of type $\{\{22/25, 3/25, 0, 0, 0, 0, 0, 0\}, \{22/25, a, 1/6 (3/25-a), 1/6 (3/25-a), 1/6 (3/25-a), 1/6 (3/25-a), 1/6 (3/25-a), 1/6 (3/25-a)\}\}$ has maximum value equal to $\{1.50589 \cdot 10^{-6}, \{a > 0.\}\}$

Point number 8 of type $\{\{22/25, 3/25, 0, 0, 0, 0, 0, 0\}, \{22/25, 0, 1/50, 1/50, 1/50, 1/50, 1/50, 1/50\}\}$ has maximum value equal to $1.5058944 \cdot 10^{-6}$

Point number 9 of type $\{\{22/25, 3/25, 0, 0, 0, 0, 0, 0\}, \{22/25, 0, 0, 3/125, 3/125, 3/125, 3/125, 3/125\}\}$ has maximum value equal to $6.05414227968 \cdot 10^{-7}$

Max attained in point 1 with value $1.580328650055674 \cdot 10^{-6}$.

EXACT VALUE OF M_4 (Unbalanced-Unbalanced distributions on different coordinates)

Point number 1 of type $\{\{22/25, a, a, a, a, a, 0\}, \{0, d, d, d, d, d, 22/25\}\}$ has maximum value equal to $\{0.0000219783, \{a > 0.02, d > 0.02\}\}$

Point number 2 of type $\{\{22/25, 0, a, a, a, a, 0\}, \{0, 3/25-5 d, d, d, d, d, 22/25\}\}$ has maximum value equal to $\{0.000015531, \{a > 0.024, d > 0.019932\}\}$

Point number 3 of type $\{\{22/25, 0, a, a, a, a, 3/25-4 a, 0\}, \{0, 3/25-4 d, d, d, d, d, 0, 22/25\}\}$ has maximum value equal to $\{9.12417 \cdot 10^{-6}, \{a > 0.0238706, d > 0.0238706\}\}$

Point number 4 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1/50, 1/50, 1/50, 1/50, 1/50, 1/50, 22/25\}\}$ has maximum value equal to 0.0000122112

Point number 5 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 3/125, 3/125, 3/125, 3/125, 3/125, 22/25\}\}$ has maximum value equal to $5.0451185664 \cdot 10^{-6}$

Max attained in point 1 with value 0.0000219783168 .

UPPER BOUND ON THE RATE

The bound on the rate for $b=8$, $k=8$ and $\epsilon=3/25$ is 0.01888868294114402 and $\eta_0=0.90706581968557$