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Title: Mathematica Output for the upper bound when $B=11$, $K=10$ and $\epsilon=1/11$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{a,a,a,a,a,a,a,a,a,a\},\{d,d,d,d,d,d,d,d,d\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.0169286,\{a>0.0909091,d>0.0909091\}\}$

Point number 2 of type $\{0,a,a,a,a,a,a,a,a,a\},\{1-10\ d,d,d,d,d,d,d,d,d\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.0168244,\{a>0.1,d>0.00909091\}\}$

Point number 3 of type $\{0,a,a,a,a,a,a,a,a,1-9\ a\},\{1-9\ d,d,d,d,d,d,d,d,d,0\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.0168245,\{a>0.010101,d>0.099969\}\}$

Point number 4 of type $\{0,0,a,a,a,a,a,a,a,a\},\{1/2\ (1-9\ d),1/2\ (1-9\ d),d,d,d,d,d,d,d,d\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.0111207,\{a>0.111111,d>0.0856433\}\}$

Point number 5 of type $\{0,0,a,a,a,a,a,a,a,1-8\ a\},\{1/2\ (1-8\ d),1/2\ (1-8\ d),d,d,d,d,d,d,d,d,0\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.0168245,\{a>0.0113636,d>0.0999301\}\}$

Point number 6 of type $\{0,0,a,a,a,a,a,a,1/2\ (1-7\ a),1/2\ (1-7\ a)\},\{1/2\ (1-7\ d),1/2\ (1-7\ d),d,d,d,d,d,d,d,d,0,0\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.00774882,\{a>0.012987,d>0.111061\}\}$

Point number 7 of type $\{0,0,0,a,a,a,a,a,a,a\},\{1/3\ (1-8\ d),1/3\ (1-8\ d),1/3\ (1-8\ d),d,d,d,d,d,d,d,d\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.00937655,\{a>0.125,d>0.0853486\}\}$

Point number 8 of type $\{0,0,0,a,a,a,a,a,a,1-7\ a\},\{1/3\ (1-7\ d),1/3\ (1-7\ d),1/3\ (1-7\ d),d,d,d,d,d,d,d,d,0\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.0168246,\{a>0.012987,d>0.0998801\}\}$

Point number 9 of type $\{0,0,0,a,a,a,a,a,1/2\ (1-6\ a),1/2\ (1-6\ a)\},\{1/3\ (1-6\ d),1/3\ (1-6\ d),1/3\ (1-6\ d),d,d,d,d,d,d,d,d,0,0\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.00774877,\{a>0.0151515,d>0.111024\}\}$

Point number 10 of type $\{0,0,0,a,a,a,a,a,1/3\ (1-5\ a),1/3\ (1-5\ a),1/3\ (1-5\ a)\},\{1/3\ (1-5\ d),1/3\ (1-5\ d),1/3\ (1-5\ d),d,d,d,d,d,d,d,d,0,0,0\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.00218562,\{a>0.0181818,d>0.124994\}\}$

Point number 11 of type $\{10/11,a,a,a,a,a,a,a,a,a\},\{0,d,d,d,d,d,d,d,d\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.0168244,\{a>0.00909091,d>0.1\}\}$

Point number 12 of type $\{10/11,a,a,a,a,a,a,a,a,1/11-9\ a\},\{0,d,d,d,d,d,d,d,d,0\}$ with $\{a,d\} \leq 1/11$ has maximum value equal to $\{0.00842991,\{a>0.,d>0.111111\}\}$

Point number 13 of type $\{10/11,a,a,a,a,a,a,a,1/2\ (1/11-8\ a),1/2\ (1/11-8\ a)\},$

{0,d,d,d,d,d,d,d,0,0}} with {a,d} $\leq 1/11$ has maximum value equal to {0.00240326,{a->0.,d->0.125}}

Point number 14 of type {{10/11,0,a,a,a,a,a,a,a,a},{0,1-9 d,d,d,d,d,d,d,d}} with {a,d} $\leq 1/11$ has maximum value equal to {0.0168245,{a->0.010101,d->0.099969}}

Point number 15 of type {{10/11,0,a,a,a,a,a,a,a,1/11-8 a},{0,1-8 d,d,d,d,d,d,d,d,0}} with {a,d} $\leq 1/11$ has maximum value equal to {0.00842991,{a->0.,d->0.111111}}

Point number 16 of type {{10/11,0,a,a,a,a,a,a,a,1/2 (1/11-7 a),1/2 (1/11-7 a)},{0,1-7 d,d,d,d,d,d,d,d,0,0}} with {a,d} $\leq 1/11$ has maximum value equal to {0.00240326,{a->0.,d->0.125}}

Point number 17 of type {{10/11,0,0,a,a,a,a,a,a,a},{0,1/2 (1-8 d),1/2 (1-8 d),d,d,d,d,d,d,d,d}} with {a,d} $\leq 1/11$ has maximum value equal to {0.0168245,{a->0.0113636,d->0.0999301}}

Point number 18 of type {{10/11,0,0,a,a,a,a,a,a,a,1/11-7 a},{0,1/2 (1-7 d),1/2 (1-7 d),d,d,d,d,d,d,d,d,0}} with {a,d} $\leq 1/11$ has maximum value equal to {0.00842991,{a->0.,d->0.111111}}

Point number 19 of type {{10/11,0,0,a,a,a,a,a,a,a,1/2 (1/11-6 a),1/2 (1/11-6 a)},{0,1/2 (1-6 d),1/2 (1-6 d),d,d,d,d,d,d,d,d,0,0}} with {a,d} $\leq 1/11$ has maximum value equal to {0.00240326,{a->0.,d->0.125}}

Point number 20 of type {{10/11,0,0,0,a,a,a,a,a,a,a},{0,1/3 (1-7 d),1/3 (1-7 d),1/3 (1-7 d),d,d,d,d,d,d,d,d}} with {a,d} $\leq 1/11$ has maximum value equal to {0.0168246,{a->0.012987,d->0.0998801}}

Point number 21 of type {{10/11,0,0,0,a,a,a,a,a,a,a,1/11-6 a},{0,1/3 (1-6 d),1/3 (1-6 d),1/3 (1-6 d),d,d,d,d,d,d,d,d,0}} with {a,d} $\leq 1/11$ has maximum value equal to {0.00842991,{a->0.,d->0.111111}}

Point number 22 of type {{10/11,0,0,0,a,a,a,a,a,a,a,1/2 (1/11-5 a),1/2 (1/11-5 a)},{0,1/3 (1-5 d),1/3 (1-5 d),1/3 (1-5 d),d,d,d,d,d,d,d,d,0,0}} with {a,d} $\leq 1/11$ has maximum value equal to {0.00240326,{a->0.,d->0.125}}

Point number 23 of type {{10/11,a,a,a,a,a,a,a,a,0},{0,d,d,d,d,d,d,d,d,10/11}} with {a,d} $\leq 1/11$ has maximum value equal to {2.63851*10⁻⁸,{a->0.010101,d->0.010101}}

Point number 24 of type {{10/11,0,a,a,a,a,a,a,a,0},{0,1/11-8 d,d,d,d,d,d,d,d,10/11}} with {a,d} $\leq 1/11$ has maximum value equal to {1.98713*10⁻⁸,{a->0.0113636,d->0.0100872}}

Point number 25 of type {{10/11,0,a,a,a,a,a,a,a,1/11-7 a,0},{0,1/11-7 d,d,d,d,d,d,d,d,10/11}} with {a,d} $\leq 1/11$ has maximum value equal to {1.33735*10⁻⁸,{a->0.0113404,d->0.0113404}}

Point number 26 of type $\{\{10/11, 0, 0, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/11-7 d), 1/2 (1/11-7 d), d, d, d, d, d, d, 10/11\}\}$ with $\{a, d\} \leq 1/11$ has maximum value equal to $\{1.53154 \cdot 10^{-8}, \{a > 0.012987, d > 0.0100839\}\}$

Point number 27 of type $\{\{10/11, 0, 0, a, a, a, a, a, 1/11-6 a, 0\}, \{0, 1/2 (1/11-6 d), 1/2 (1/11-6 d), d, d, d, d, d, d, 0, 10/11\}\}$ with $\{a, d\} \leq 1/11$ has maximum value equal to $\{8.82623 \cdot 10^{-9}, \{a > 0.0129138, d > 0.0113424\}\}$

Point number 28 of type $\{\{10/11, 0, 0, a, a, a, a, 1/2 (1/11-5 a), 1/2 (1/11-5 a), 0\}, \{0, 1/2 (1/11-5 d), 1/2 (1/11-5 d), d, d, d, d, d, 0, 0, 10/11\}\}$ with $\{a, d\} \leq 1/11$ has maximum value equal to $\{4.27193 \cdot 10^{-9}, \{a > 0.0129352, d > 0.0129352\}\}$

Point number 29 of type $\{\{10/11, 1/11, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, a, (1-a)/9, (1-a)/9, (1-a)/9, (1-a)/9, (1-a)/9, (1-a)/9, (1-a)/9, (1-a)/9\}\}$ with $\{a\} \leq 1/11$ has maximum value equal to $\{0.0168282, \{a > 0.0974212\}\}$

Point number 30 of type $\{\{10/11, 1/11, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9\}\}$ with $\{a\} \leq 1/11$ has maximum value equal to $0.008429910375751965`$

Point number 31 of type $\{\{10/11, 1/11, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8\}\}$ with $\{a\} \leq 1/11$ has maximum value equal to $0.00240325927734375`$

Max attained in point 1 with value $0.016928619813050806`$.

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a, a, a, a, a, a, a, a, a, a\}, \{d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.0169286, \{a > 0.0909091, d > 0.0909091\}\}$

Point number 2 of type $\{\{0, a, a, a, a, a, a, a, a, a\}, \{1-10 d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.018144, \{a > 0.1, d > 0.\}\}$

Point number 3 of type $\{\{0, a, a, a, a, a, a, a, a, 1-9 a\}, \{1-9 d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.018144, \{a > 0., d > 0.1\}\}$

Point number 4 of type $\{\{0, 0, a, a, a, a, a, a, a, a\}, \{1/2 (1-9 d), 1/2 (1-9 d), d, d, d, d, d, d, d, d\}\}$ with value $\{0.0111207, \{a > 0.111111, d > 0.0856433\}\}$

Point number 5 of type $\{\{0, 0, a, a, a, a, a, a, a, 1-8 a\}, \{1/2 (1-8 d), 1/2 (1-8 d), d, d, d, d, d, d, d, 0\}\}$ with value $\{0.018144, \{a > 0., d > 0.1\}\}$

Point number 6 of type $\{\{0, 0, a, a, a, a, a, a, 1/2 (1-7 a), 1/2 (1-7 a)\}, \{1/2 (1-7 d), 1/2 (1-7 d), d, d, d, d, d, d, 0, 0\}\}$ with value $\{0.00842991, \{a > 0., d > 0.111111\}\}$

Point number 7 of type $\{\{0, 0, 0, a, a, a, a, a, a, a\}, \{1/3 (1-8 d), 1/3 (1-8 d), 1/3 (1-8 d), d, d, d, d, d, d, d\}\}$ with value $\{0.00937655, \{a > 0.125, d > 0.0853486\}\}$

Point number 8 of type $\{\{0, 0, 0, a, a, a, a, a, a, 1-7 a\}, \{1/3 (1-7 d), 1/3 (1-7 d), 1/3 (1-7 d), d, d, d, d, d, d, d\}\}$

d),d,d,d,d,d,d,d,0}} with value {0.018144,{a->0.,d->0.1}}

Point number 9 of type {{0,0,0,a,a,a,a,a,1/2 (1-6 a),1/2 (1-6 a)},1/3 (1-6 d),1/3 (1-6 d),1/3 (1-6 d),d,d,d,d,d,d,0,0}} with value {0.00842991,{a->0.,d->0.111111}}

Point number 10 of type {{0,0,0,a,a,a,a,a,1/3 (1-5 a),1/3 (1-5 a),1/3 (1-5 a)},1/3 (1-5 d),1/3 (1-5 d),1/3 (1-5 d),d,d,d,d,d,d,0,0,0}} with value {0.00240326,{a->0.,d->0.125}}

Max attained in point 2 with value 0.018144`.

EXACT VALUE OF M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type {{10/11,a,a,a,a,a,a,a,a,a},{10/11,d,d,d,d,d,d,d,d,d}} has maximum value equal to {1.38815*10⁻⁹,{a->0.00909091,d->0.00909091}}

Point number 2 of type {{10/11,0,a,a,a,a,a,a,a,a},{10/11,1/11-9 d,d,d,d,d,d,d,d,d}} has maximum value equal to {1.32636*10⁻⁹,{a->0.010101,d->0.}}

Point number 3 of type {{10/11,0,a,a,a,a,a,a,a,1/11-8 a},{10/11,1/11-8 d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {1.32636*10⁻⁹,{a->0.,d->0.010101}}

Point number 4 of type {{10/11,0,0,a,a,a,a,a,a,a},{10/11,1/2 (1/11-8 d),1/2 (1/11-8 d),d,d,d,d,d,d,d,d}} has maximum value equal to {9.3031*10⁻¹⁰,{a->0.0113636,d->0.00848067}}

Point number 5 of type {{10/11,0,0,a,a,a,a,a,a,a,1/11-7 a},{10/11,1/2 (1/11-7 d),1/2 (1/11-7 d),d,d,d,d,d,d,d,0}} has maximum value equal to {1.32636*10⁻⁹,{a->0.,d->0.010101}}

Point number 6 of type {{10/11,0,0,a,a,a,a,a,a,1/2 (1/11-6 a),1/2 (1/11-6 a)},10/11,1/2 (1/11-6 d),1/2 (1/11-6 d),d,d,d,d,d,d,0,0}} has maximum value equal to {6.6351*10⁻¹⁰,{a->0.,d->0.0113636}}

Point number 7 of type {{10/11,0,0,0,a,a,a,a,a,a},{10/11,1/3 (1/11-7 d),1/3 (1/11-7 d),1/3 (1/11-7 d),d,d,d,d,d,d,d}} has maximum value equal to {7.78919*10⁻¹⁰,{a->0.012987,d->0.00842695}}

Point number 8 of type {{10/11,0,0,0,a,a,a,a,a,1/11-6 a},{10/11,1/3 (1/11-6 d),1/3 (1/11-6 d),1/3 (1/11-6 d),d,d,d,d,d,d,0}} has maximum value equal to {1.32636*10⁻⁹,{a->0.,d->0.010101}}

Point number 9 of type {{10/11,0,0,0,a,a,a,a,a,1/2 (1/11-5 a),1/2 (1/11-5 a)},10/11,1/3 (1/11-5 d),1/3 (1/11-5 d),1/3 (1/11-5 d),d,d,d,d,d,d,0,0}} has maximum value equal to {6.6351*10⁻¹⁰,{a->0.,d->0.0113636}}

Point number 10 of type {{10/11,0,0,0,a,a,a,a,1/3 (1/11-4 a),1/3 (1/11-4 a),1/3 (1/11-4 a)},10/11,1/3 (1/11-4 d),1/3 (1/11-4 d),1/3 (1/11-4 d),d,d,d,d,d,d,0,0,0}} has maximum value equal to {2.07635*10⁻¹⁰,{a->0.,d->0.012987}}

Point number 11 of type $\{\{10/11, 1/11, 0, 0, 0, 0, 0, 0, 0, 0\}, \{10/11, a, 1/9 (1/11-a), 1/9 (1/11-a), 1/9 (1/11-a), 1/9 (1/11-a), 1/9 (1/11-a), 1/9 (1/11-a), 1/9 (1/11-a), 1/9 (1/11-a)\}\}$ has maximum value equal to $\{1.32636 \cdot 10^{-9}, \{a > 0.\}\}$

Point number 12 of type $\{\{10/11, 1/11, 0, 0, 0, 0, 0, 0, 0, 0\}, \{10/11, 0, 1/99, 1/99, 1/99, 1/99, 1/99, 1/99, 1/99, 1/99\}\}$ has maximum value equal to $1.3263639228899158 \cdot 10^{-9}$

Point number 13 of type $\{\{10/11, 1/11, 0, 0, 0, 0, 0, 0, 0, 0\}, \{10/11, 0, 0, 1/88, 1/88, 1/88, 1/88, 1/88, 1/88, 1/88, 1/88\}\}$ has maximum value equal to $6.635099648403444 \cdot 10^{-10}$

Point number 14 of type $\{\{10/11, 1/11, 0, 0, 0, 0, 0, 0, 0, 0\}, \{10/11, 0, 0, 0, 1/77, 1/77, 1/77, 1/77, 1/77, 1/77, 1/77\}\}$ has maximum value equal to $2.0763476798149692 \cdot 10^{-10}$

Max attained in point 1 with value $1.3881468246701661 \cdot 10^{-9}$.

EXACT VALUE OF M₄ (Unbalanced-Unbalanced distributions on different coordinates)

Point number 1 of type $\{\{10/11, a, a, a, a, a, a, a, a, 0\}, \{0, d, d, d, d, d, d, d, d, 10/11\}\}$ has maximum value equal to $\{2.63851 \cdot 10^{-8}, \{a > 0.010101, d > 0.010101\}\}$

Point number 2 of type $\{\{10/11, 0, a, a, a, a, a, a, a, 0\}, \{0, 1/11-8 d, d, d, d, d, d, d, d, 10/11\}\}$ has maximum value equal to $\{1.98713 \cdot 10^{-8}, \{a > 0.0113636, d > 0.0100872\}\}$

Point number 3 of type $\{\{10/11, 0, a, a, a, a, a, a, 1/11-7 a, 0\}, \{0, 1/11-7 d, d, d, d, d, d, d, 0, 10/11\}\}$ has maximum value equal to $\{1.33735 \cdot 10^{-8}, \{a > 0.0113404, d > 0.0113404\}\}$

Point number 4 of type $\{\{10/11, 0, 0, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/11-7 d), 1/2 (1/11-7 d), d, d, d, d, d, d, 10/11\}\}$ has maximum value equal to $\{1.53154 \cdot 10^{-8}, \{a > 0.012987, d > 0.0100839\}\}$

Point number 5 of type $\{\{10/11, 0, 0, a, a, a, a, a, 1/11-6 a, 0\}, \{0, 1/2 (1/11-6 d), 1/2 (1/11-6 d), d, d, d, d, d, 0, 10/11\}\}$ has maximum value equal to $\{8.82623 \cdot 10^{-9}, \{a > 0.0129138, d > 0.0113424\}\}$

Point number 6 of type $\{\{10/11, 0, 0, a, a, a, a, 1/2 (1/11-5 a), 1/2 (1/11-5 a), 0\}, \{0, 1/2 (1/11-5 d), 1/2 (1/11-5 d), d, d, d, d, d, 0, 0, 10/11\}\}$ has maximum value equal to $\{4.27193 \cdot 10^{-9}, \{a > 0.0129352, d > 0.0129352\}\}$

Point number 7 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1/99, 1/99, 1/99, 1/99, 1/99, 1/99, 1/99, 1/99, 10/11\}\}$ has maximum value equal to $1.4196741611309584 \cdot 10^{-8}$

Point number 8 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/88, 1/88, 1/88, 1/88, 1/88, 1/88, 1/88, 10/11\}\}$ has maximum value equal to

$7.186495794299951 \times 10^{-9}$

Point number 9 of type $\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$,
 $\{0, 0, 0, 1/77, 1/77, 1/77, 1/77, 1/77, 1/77, 10/11\}$ has maximum value equal to
 $2.2839824477964663 \times 10^{-9}$

Max attained in point 1 with value $2.638506872901886 \times 10^{-8}$.

UPPER BOUND ON THE RATE

The bound on the rate for $b=11$, $k=10$ and $\epsilon=1/11$ is 0.013203218287013135
and $\eta_0=0.9372200104168513$