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Title: Mathematica Output for the upper bound when $B=K=5$ and $\epsilon=1/44(4+\sqrt{5})$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,a,a\},\{d,d,d,d,d\}\}$ with $\{a,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.384,\{a \rightarrow 0.2,d \rightarrow 0.2\}\}$

Point number 2 of type $\{\{1/44 (4+\sqrt{5}),a,a,a,a\},\{c,d,d,d,d\}\}$ with $\{a,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.384033,\{a \rightarrow 0.214568,c \rightarrow 0.2589,d \rightarrow 0.185275\}\}$

Point number 3 of type $\{\{1/44 (4+\sqrt{5}),a,a,a,b\},\{c,d,d,d,1/44 (4+\sqrt{5})\}\}$ with $\{a,b,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.384,\{a \rightarrow 0.2,b \rightarrow 0.258271,c \rightarrow 0.258271,d \rightarrow 0.2\}\}$

Point number 4 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),a,a,a\},\{c,c,d,d,d\}\}$ with $\{a,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.383609,\{a \rightarrow 0.238847,c \rightarrow 0.255545,d \rightarrow 0.16297\}\}$

Point number 5 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),a,a,b\},\{c,c,d,d,1/44 (4+\sqrt{5})\}\}$ with $\{a,b,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.383605,\{a \rightarrow 0.228182,b \rightarrow 0.260179,c \rightarrow 0.255481,d \rightarrow 0.173655\}\}$

Point number 6 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),a,b,b\},\{c,c,d,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with $\{a,b,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.383453,\{a \rightarrow 0.201301,b \rightarrow 0.257621,c \rightarrow 0.257621,d \rightarrow 0.201301\}\}$

Point number 7 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),a,a\},\{c,c,c,d,d\}\}$ with $\{a,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.380985,\{a \rightarrow 0.287407,c \rightarrow 0.238847,d \rightarrow 0.141729\}\}$

Point number 8 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),a,b\},\{c,c,c,d,1/44 (4+\sqrt{5})\}\}$ with $\{a,b,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.380985,\{a \rightarrow 0.287407,b \rightarrow 0.287407,c \rightarrow 0.238847,d \rightarrow 0.141729\}\}$

Point number 9 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),b,b\},\{c,c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with $\{b,c\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.380985,\{b \rightarrow 0.287407,c \rightarrow 0.238847\}\}$

Point number 10 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),b,b\},\{c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with $\{b,c\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.369205,\{b \rightarrow 0.287407,c \rightarrow 0.287407\}\}$

Point number 11 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),a\},\{c,c,c,c,d\}\}$ with $\{a,c,d\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.361605,\{a \rightarrow 0.433085,c \rightarrow 0.214568,d \rightarrow 0.141729\}\}$

Point number 12 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44$

$(4+\sqrt{5}),b\},\{c,c,c,c,1/44 (4+\sqrt{5})\}$ with $\{b,c\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.361605,\{b->0.433085,c->0.214568\}\}$

Point number 13 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),b\},\{c,c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with $\{b,c\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.355123,\{b->0.433085,c->0.238847\}\}$

Point number 14 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),b\},\{c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with $\{b,c\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.343343,\{b->0.433085,c->0.287407\}\}$

Point number 15 of type $\{\{1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),b\},\{c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with $\{b,c\} \geq 1/44 (4+\sqrt{5})$ has maximum equal to $\{0.317481,\{b->0.433085,c->0.433085\}\}$

Max attained in point 2 with value 0.3840327769373233`.

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,b\},\{c,c,c,c,1/44 (4+\sqrt{5})\}\}$ with conditions $\{a \leq 1/44 (4+\sqrt{5}),b \leq 1,c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.361605,\{a->0.141729,b->0.433085,c->0.214568\}\}$

Point number 2 of type $\{\{a,a,a,b\},\{c,c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with conditions $\{a \leq 1/44 (4+\sqrt{5}),b \leq 1,c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.380985,\{a->0.141729,b->0.287407,c->0.238847\}\}$

Point number 3 of type $\{\{a,a,b,b\},\{c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with conditions $\{a \leq 1/44 (4+\sqrt{5}),b \leq 1,c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.382127,\{a->0.121146,b->0.252569,c->0.287407\}\}$

Point number 4 of type $\{\{a,b,b,b\},\{c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with conditions $\{a \leq 1/44 (4+\sqrt{5}),b \leq 1,c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.389226,\{a->0.,b->0.25,c->0.433085\}\}$

Point number 5 of type $\{\{1/44 (4+\sqrt{5}),a,a,a\},\{c,d,d,d\}\}$ with conditions $\{a \leq 1,c \geq 1/44 (4+\sqrt{5}),d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.384033,\{a->0.214568,c->0.2589,d->0.185275\}\}$

Point number 6 of type $\{\{1/44 (4+\sqrt{5}),a,a,a,b\},\{c,d,d,d,1/44 (4+\sqrt{5})\}\}$ with conditions $\{a \leq 1,b \leq 1,c \geq 1/44 (4+\sqrt{5}),d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.384,\{a->0.2,b->0.258271,c->0.258271,d->0.2\}\}$

Point number 7 of type $\{\{1/44 (4+\sqrt{5}),a,a,b,b\},\{c,d,d,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$ with conditions $\{a \leq 1,b \leq 1,c \geq 1/44 (4+\sqrt{5}),d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.383605,\{a->0.173655,b->0.255481,c->0.260179,d->0.228182\}\}$

Point number 8 of type $\{\{1/44 (4+\sqrt{5}),a,b,b,b\},\{c,d,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}\}$

$(4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.382394, \{a > 0.0891618, b > 0.25637, c > 0.25637, d > 0.318444\}\}$

Point number 9 of type $\{1/44 (4+\sqrt{5}), b, b, b, b\}, \{c, 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{b \leq 1, c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.361605, \{b > 0.214568, c > 0.433085\}\}$

Point number 10 of type $\{0, a, a, a, a\}, \{c, d, d, d, d\}$ with conditions $\{a \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.389226, \{a > 0.25, c > 0.433085, d > 0.141729\}\}$

Point number 11 of type $\{0, a, a, a, b\}, \{c, d, d, d, 1/44 (4+\sqrt{5})\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.389226, \{a > 0.25, b > 0.25, c > 0.433085, d > 0.141729\}\}$

Point number 12 of type $\{0, a, a, b, b\}, \{c, d, d, 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.389226, \{a > 0.25, b > 0.25, c > 0.433085, d > 0.141729\}\}$

Point number 13 of type $\{0, a, b, b, b\}, \{c, d, 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.389226, \{a > 0.25, b > 0.25, c > 0.433085, d > 0.141729\}\}$

Point number 14 of type $\{0, b, b, b, b\}, \{c, 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{b \leq 1, c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.389226, \{b > 0.25, c > 0.433085\}\}$

Point number 15 of type $\{0, 0, a, a, a\}, \{c, c, d, d, d\}$ with conditions $\{a \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.3375, \{a > 0.333333, c > 0.287407, d > 0.141729\}\}$

Point number 16 of type $\{0, 0, a, a, b\}, \{c, c, d, d, 1/44 (4+\sqrt{5})\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.3375, \{a > 0.333333, b > 0.333333, c > 0.287407, d > 0.141729\}\}$

Point number 17 of type $\{0, 0, a, b, b\}, \{c, c, d, 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.3375, \{a > 0.333333, b > 0.333333, c > 0.287407, d > 0.141729\}\}$

Point number 18 of type $\{0, 0, b, b, b\}, \{c, c, 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{b \leq 1, c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.3375, \{b > 0.333333, c > 0.287407\}\}$

Point number 19 of type $\{0, 0, b, b, b\}, \{c, 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5}), 1/44 (4+\sqrt{5})\}$ with conditions $\{b \leq 1, c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.301407, \{b > 0.333333, c > 0.433085\}\}$

Point number 20 of type $\{0, 0, 0, 0, 1\}, \{c, c, c, c, d\}$ with conditions $\{c \geq 1/44 (4+\sqrt{5}), d \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.237085, \{c > 0.214568, d > 0.141729\}\}$

$>0.141729\}$

Point number 21 of type $\{0,0,0,0,1\},\{c,c,c,c,1/44 (4+\sqrt{5})\}$ with conditions $\{c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.237085, \{c > 0.214568\}\}$

Point number 22 of type $\{0,0,0,0,1\},\{c,c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}$ with conditions $\{c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.227291, \{c > 0.238847\}\}$

Point number 23 of type $\{0,0,0,0,1\},\{c,c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}$ with conditions $\{c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.209764, \{c > 0.287407\}\}$

Point number 24 of type $\{0,0,0,0,1\},\{c,1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5}),1/44 (4+\sqrt{5})\}$ with conditions $\{c \geq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.173671, \{c > 0.433085\}\}$

Max attained in point 4 with value 0.38922573329674426`.

UPPER BOUND ON M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type $\{s_1,(1-s_1)/4,(1-s_1)/4,(1-s_1)/4,(1-s_1)/4\},\{s_2,(1-s_2)/4,(1-s_2)/4,(1-s_2)/4,(1-s_2)/4\}$ with conditions $\{s_1 \leq 1/44 (4+\sqrt{5}), s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.370556, \{s_1 > 0.141729, s_2 > 0.141729\}\}$

Point number 2 of type $\{s_1,a,a,a,b\},\{s_2,(1-s_2)/3,(1-s_2)/3,(1-s_2)/3,0\}$ with conditions $\{s_1 \leq 1/44 (4+\sqrt{5}), s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.374758, \{s_1 > 0.141729, s_2 > 0.141729, a > 0.134696, b > 0.454183\}\}$

Point number 3 of type $\{s_1,a,a,b,b\},\{s_2,(1-s_2)/2,(1-s_2)/2,0,0\}$ with conditions $\{s_1 \leq 1/44 (4+\sqrt{5}), s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.310309, \{s_1 > 0.141729, s_2 > 0.141729, a > 0.0916948, b > 0.337441\}\}$

Point number 4.1 of type $\{a,0,a,a,b\},\{d,c,d,d,0\}$ with conditions $\{a \leq 1/44 (4+\sqrt{5}), d \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.317481, \{a > 0.141729, b > 0.574814, c > 0.574814, d > 0.141729\}\}$

Point number 4.2.1 of type $\{0,0,a,a,b\},\{c,c,d,d,0\}$ with conditions $\{c \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.304664, \{a > 0., b > 1., c > 0.141729, d > 0.358271\}\}$

Point number 4.2.2 of type $\{0,0,a,a,b\},\{0,c,d,d,0\}$ with conditions $\{ \}$ has maximum equal to $\{0.222222, \{a > 0., b > 1., c > 0.333333, d > 0.333333\}\}$

Point number 4.2.3 of type $\{0,0,a,a,b\},\{1/44 (4+\sqrt{5}),c,d,d,0\}$ with conditions $\{ \}$ has maximum equal to $\{0.349298, \{a > 0., b > 1., c > 0.28609, d > 0.28609\}\}$

Point number 4.3.1 of type $\{1/44 (4+\sqrt{5}),0,0,0,b\},\{s_2,c,c,c,0\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.319705, \{s_2 > 0.141729, b > 0.858271, c > 0.28609\}\}$

Point number 4.3.2 of type $\{\{1/44 (4+\sqrt{5}), 0, 1/2 (1+1/44 (-4-\sqrt{5})), 1/2 (1+1/44 (-4-\sqrt{5})), 0\}, \{s_2, c, d, 0\}\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.159298, \{s_2 > 0.141729, c > 0.572181, d > 0.143045\}\}$

Point number 4.3.3 of type $\{\{1/44 (4+\sqrt{5}), 0, a, a, b\}, \{s_2, 0, (1-s_2)/2, (1-s_2)/2, 0\}\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.159298, \{s_2 > 0.141729, a > 0.143045, b > 0.572181\}\}$

Point number 4.3.4 of type $\{\{1/44 (4+\sqrt{5}), 0, 1/3 (1+1/44 (-4-\sqrt{5})), 1/3 (1+1/44 (-4-\sqrt{5})), 1/3 (1+1/44 (-4-\sqrt{5}))\}, \{s_2, 1-s_2, 0, 0, 0\}\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.349298, \{s_2 > 0.\}\}$

Point number 4.3.5 of type $\{\{1/44 (4+\sqrt{5}), 0, a, a, b\}, \{1/44 (4+\sqrt{5}), c, d, d, 0\}\}$ with conditions $\{\}$ has maximum equal to $\{0.370556, \{a > 0.214568, b > 0.429136, c > 0.429136, d > 0.214568\}\}$

Point number 4.3.6 of type $\{\{1/44 (4+\sqrt{5}), 0, a, a, b\}, \{0, c, d, d, 0\}\}$ with conditions $\{\}$ has maximum equal to $\{0.349298, \{a > 0.28609, b > 0.28609, c > 1., d > 0.\}\}$

Point number 5.1 of type $\{\{a, 0, a, b, b\}, \{d, c, d, 0, 0\}\}$ with conditions $\{a \leq 1/44 (4+\sqrt{5}), d \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.311125, \{a > 0.141729, b > 0.358271, c > 0.716542, d > 0.141729\}\}$

Point number 5.2.1 of type $\{\{0, 0, a, b, b\}, \{c, c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum equal to $\{0.316406, \{a > 0.25, b > 0.375, c > 0.375, d > 0.25\}\}$

Point number 5.2.2 of type $\{\{0, 0, a, b, b\}, \{0, c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum equal to $\{0.222222, \{a > 0.333333, b > 0.333333, c > 1., d > 0.\}\}$

Point number 5.2.3 of type $\{\{0, 0, a, b, b\}, \{1/44 (4+\sqrt{5}), c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum equal to $\{0.256403, \{a > 0.279856, b > 0.360072, c > 0.606884, d > 0.251387\}\}$

Point number 5.3.1 of type $\{\{1/44 (4+\sqrt{5}), 0, 0, b, b\}, \{s_2, c, c, 0, 0\}\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.268815, \{s_2 > 0.141729, b > 0.429136, c > 0.429136\}\}$

Point number 5.3.2 of type $\{\{1/44 (4+\sqrt{5}), 0, a, 0, 0\}, \{s_2, c, d, 0, 0\}\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0., \{s_2 > 0., a > 0.858271, c > 1., d > 0.\}\}$

Point number 5.3.3 of type $\{\{1/44 (4+\sqrt{5}), 0, a, b, b\}, \{s_2, 0, d, 0, 0\}\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.156602, \{s_2 > 0., a > 0., b > 0.429136, d > 1.\}\}$

Point number 5.3.4 of type $\{\{1/44 (4+\sqrt{5}), 0, a, a, a\}, \{s_2, c, 0, 0, 0\}\}$ with conditions $\{s_2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.349298, \{s_2 > 0., a > 0.28609, c > 1.\}\}$

Point number 5.3.5 of type $\{\{1/44 (4+\sqrt{5}), 0, a, b, b\}, \{1/44 (4+\sqrt{5}), c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum equal to $\{0.332372, \{a > 0.238175, b > 0.310048, c > 0.310048, d > 0.310048\}\}$

$>0.678482, d->0.179789\}$

Point number 5.3.6 of type $\{\{1/44 (4+\sqrt{5}), 0, a, b, b\}, \{0, c, d, 0, 0\}\}$ with conditions $\{$ has maximum equal to $\{0.349298, \{a->0.28609, b->0.28609, c->1., d->0.\}\}$

Point number 6.1 of type $\{\{a, 0, 0, b, b\}, \{d, c, c, 0, 0\}\}$ with conditions $\{a \leq 1/44 (4+\sqrt{5}), d \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.268815, \{a->0.141729, b->0.429136, c->0.429136, d->0.141729\}\}$

Point number 6.2.1 of type $\{\{0, 0, 0, b, b\}, \{c, c, c, 0, 0\}\}$ with conditions $\{$ has maximum equal to $\{0.222222, \{b->0.5, c->0.333333\}\}$

Point number 6.2.2 of type $\{\{0, 0, 0, b, b\}, \{0, c, c, 0, 0\}\}$ with conditions $\{$ has maximum equal to $\{0., \{b->0.5, c->0.5\}\}$

Point number 6.2.3 of type $\{\{0, 0, 0, b, b\}, \{1/44 (4+\sqrt{5}), c, c, 0, 0\}\}$ with conditions $\{$ has maximum equal to $\{0.156602, \{b->0.5, c->0.429136\}\}$

Point number 6.3.1 of type $\{\{1/44 (4+\sqrt{5}), 0, 0, b, b\}, \{s^2, c, c, 0, 0\}\}$ with conditions $\{s^2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.268815, \{s^2->0.141729, b->0.429136, c->0.429136\}\}$

Point number 6.3.2 of type $\{\{1/44 (4+\sqrt{5}), 0, 0, b, b\}, \{s^2, c, c, 0, 0\}\}$ with conditions $\{s^2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.268815, \{s^2->0.141729, b->0.429136, c->0.429136\}\}$

Point number 6.3.3 of type $\{\{1/44 (4+\sqrt{5}), 0, 0, b, b\}, \{s^2, c, c, 0, 0\}\}$ with conditions $\{s^2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.268815, \{s^2->0.141729, b->0.429136, c->0.429136\}\}$

Point number 6.3.4 of type $\{\{1/44 (4+\sqrt{5}), 0, 0, a, a\}, \{s^2, c, c, 0, 0\}\}$ with conditions $\{s^2 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.268815, \{s^2->0.141729, a->0.429136, c->0.429136\}\}$

Point number 6.3.5 of type $\{\{1/44 (4+\sqrt{5}), 0, 0, b, b\}, \{1/44 (4+\sqrt{5}), c, c, 0, 0\}\}$ with conditions $\{$ has maximum equal to $\{0.268815, \{b->0.429136, c->0.429136\}\}$

Point number 6.3.6 of type $\{\{1/44 (4+\sqrt{5}), 0, 0, b, b\}, \{0, c, c, 0, 0\}\}$ with conditions $\{$ has maximum equal to $\{0.156602, \{b->0.429136, c->0.5\}\}$

Point number 7 of type $\{\{s_1, 0, (1-s_1)/3, (1-s_1)/3, (1-s_1)/3\}, \{0, 1, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.349298, \{s_1->0.141729\}\}$

Point number 8 of type $\{\{s_1, 0, 0, (1-s_1)/2, (1-s_1)/2\}, \{0, 1, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/44 (4+\sqrt{5})\}$ has maximum equal to $\{0.156602, \{s_1->0.141729\}\}$

Max attained in point 2 with value 0.3747581115851034`.

EXACT VALUE OF M_4 (Unbalanced-Unbalanced distributions on different coordinates)

Point number 1 of type $\{\{1/5, 1/5, 1/5, 1/5, 1/5\}, \{1/5, 1/5, 1/5, 1/5, 1/5\}\}$ has maximum equal to 0.384`

Point number 2 of type $\{\{0, 0, a, b, b\}, \{c, c, d, 0, 0\}\}$ has maximum equal to $\{0.316406, \{a > 0.25, b > 0.375, c > 0.375, d > 0.25\}\}$

Point number 3 of type $\{\{0, 0, a, a, b\}, \{c, c, d, d, 0\}\}$ has maximum equal to $\{0.375, \{a > 0., b > 1., c > 0.25, d > 0.25\}\}$

Point number 4 of type $\{\{0, 0, 1/3, 1/3, 1/3\}, \{c, c, d, d, d\}\}$ has maximum equal to $\{0.345679, \{c > 0.333333, d > 0.111111\}\}$

Point number 5 of type $\{\{0, a, a, a, b\}, \{c, d, d, d, 0\}\}$ has maximum equal to $\{0.384, \{a > 0.2, b > 0.4, c > 0.4, d > 0.2\}\}$

Point number 6 of type $\{\{0, 1/4, 1/4, 1/4, 1/4\}, \{c, d, d, d, d\}\}$ has maximum equal to $\{0.389226, \{c > 0.433085, d > 0.141729\}\}$

Max attained in point 6 with value 0.38922573329674415`.

UPPER BOUND ON THE RATE

The bound on the rate for $b=k=5$ and $\epsilon=1/44$ ($4+\sqrt{5}$) is 0.16893245843873225`.