

Authors: S. Della Fiore, S. Costa, M. Dalai

Title: Mathematica Output for the upper bound when $B=12$, $K=10$ and $\epsilon=1/20$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,a,a,a,a,a,a,a,a\},\{d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0309446,\{a>0.0833333,d>0.0833333\}\}$

Point number 2 of type $\{\{0,a,a,a,a,a,a,a,a,a\},\{1-11\ d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0299072,\{a>0.0909091,d->0.00454545\}\}$

Point number 3 of type $\{\{0,a,a,a,a,a,a,a,a,a,1-10\ a\},\{1-10\ d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0299073,\{a>0.005,d->0.0908907\}\}$

Point number 4 of type $\{\{0,0,a,a,a,a,a,a,a,a\},\{1/2\ (1-10\ d),1/2\ (1-10\ d),d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0221641,\{a>0.1,d->0.0785157\}\}$

Point number 5 of type $\{\{0,0,a,a,a,a,a,a,a,a,1-9\ a\},\{1/2\ (1-9\ d),1/2\ (1-9\ d),d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0299073,\{a>0.00555556,d->0.0908683\}\}$

Point number 6 of type $\{\{0,0,a,a,a,a,a,a,a,1/2\ (1-8\ a),1/2\ (1-8\ a)\},\{1/2\ (1-8\ d),1/2\ (1-8\ d),d,d,d,d,d,d,d,d,0,0\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0174183,\{a>0.00625,d->0.0999628\}\}$

Point number 7 of type $\{\{0,0,0,a,a,a,a,a,a,a\},\{1/3\ (1-9\ d),1/3\ (1-9\ d),1/3\ (1-9\ d),d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0188293,\{a>0.111111,d->0.0778177\}\}$

Point number 8 of type $\{\{0,0,0,a,a,a,a,a,a,a,1-8\ a\},\{1/3\ (1-8\ d),1/3\ (1-8\ d),1/3\ (1-8\ d),d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0299074,\{a>0.00625,d->0.0908403\}\}$

Point number 9 of type $\{\{0,0,0,a,a,a,a,a,a,1/2\ (1-7\ a),1/2\ (1-7\ a)\},\{1/3\ (1-7\ d),1/3\ (1-7\ d),1/3\ (1-7\ d),d,d,d,d,d,d,d,d,0,0\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0174183,\{a>0.00714286,d->0.0999363\}\}$

Point number 10 of type $\{\{0,0,0,a,a,a,a,a,a,1/3\ (1-6\ a),1/3\ (1-6\ a),1/3\ (1-6\ a)\},\{1/3\ (1-6\ d),1/3\ (1-6\ d),1/3\ (1-6\ d),d,d,d,d,d,d,d,d,0,0,0\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0080554,\{a>0.00833333,d->0.111065\}\}$

Point number 11 of type $\{\{0,0,0,0,a,a,a,a,a,a\},\{1/4\ (1-8\ d),1/4\ (1-8\ d),1/4\ (1-8\ d),1/4\ (1-8\ d),d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0167487,\{a>0.125,d->0.0771487\}\}$

Point number 12 of type $\{\{0,0,0,0,a,a,a,a,a,a,1-7\ a\},\{1/4\ (1-7\ d),1/4\ (1-7\ d),1/4\ (1-7\ d),1/4\ (1-7\ d),d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{0.0167487,\{a>0.125,d->0.0771487\}\}$

d), $1/4 (1-7 d)$, d, d, d, d, d, d, 0}} with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.0299074, \{a > 0.00714286, d > 0.0908042\}\}$

Point number 13 of type $\{0, 0, 0, 0, a, a, a, a, a, 1/2 (1-6 a), 1/2 (1-6 a)\}, \{1/4 (1-6 d), 1/4 (1-6 d), 1/4 (1-6 d), 1/4 (1-6 d), d, d, d, d, d, 0, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.0174183, \{a > 0.00833333, d > 0.0999008\}\}$

Point number 14 of type $\{0, 0, 0, 0, a, a, a, a, a, 1/3 (1-5 a), 1/3 (1-5 a), 1/3 (1-5 a)\}, \{1/4 (1-5 d), 1/4 (1-5 d), 1/4 (1-5 d), 1/4 (1-5 d), d, d, d, d, 0, 0, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00805532, \{a > 0.01, d > 0.111037\}\}$

Point number 15 of type $\{0, 0, 0, 0, a, a, a, a, 1/4 (1-4 a), 1/4 (1-4 a), 1/4 (1-4 a), 1/4 (1-4 a)\}, \{1/4 (1-4 d), 1/4 (1-4 d), 1/4 (1-4 d), 1/4 (1-4 d), d, d, d, d, 0, 0, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00254084, \{a > 0.111111, d > 0.111111\}\}$

Point number 16 of type $\{19/20, a, a, a, a, a, a, a, a, a, a, a, 0, d, d, d, d, d, d, d, d, d\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.0299072, \{a > 0.00454545, d > 0.0909091\}\}$

Point number 17 of type $\{19/20, a, a, a, a, a, a, a, a, a, 1/20-10 a\}, \{0, d, d, d, d, d, d, d, d, d, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.018144, \{a > 0., d > 0.1\}\}$

Point number 18 of type $\{19/20, a, a, a, a, a, a, a, a, a, 1/2 (1/20-9 a), 1/2 (1/20-9 a)\}, \{0, d, d, d, d, d, d, d, d, 0, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00842991, \{a > 0., d > 0.111111\}\}$

Point number 19 of type $\{19/20, a, a, a, a, a, a, a, a, a, 1/3 (1/20-8 a), 1/3 (1/20-8 a), 1/3 (1/20-8 a)\}, \{0, d, d, d, d, d, d, d, d, 0, 0, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00240326, \{a > 0., d > 0.125\}\}$

Point number 20 of type $\{19/20, 0, a, a, a, a, a, a, a, a, a, a, 0, 1-10 d, d, d, d, d, d, d, d, d\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.0299073, \{a > 0.005, d > 0.0908907\}\}$

Point number 21 of type $\{19/20, 0, a, a, a, a, a, a, a, a, 1/20-9 a\}, \{0, 1-9 d, d, d, d, d, d, d, d, d, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.018144, \{a > 0., d > 0.1\}\}$

Point number 22 of type $\{19/20, 0, a, a, a, a, a, a, a, a, 1/2 (1/20-8 a), 1/2 (1/20-8 a)\}, \{0, 1-8 d, d, d, d, d, d, d, d, d, 0, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00842991, \{a > 0., d > 0.111111\}\}$

Point number 23 of type $\{19/20, 0, a, a, a, a, a, a, a, a, 1/3 (1/20-7 a), 1/3 (1/20-7 a), 1/3 (1/20-7 a)\}, \{0, 1-7 d, d, d, d, d, d, d, d, 0, 0, 0\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00240326, \{a > 0., d > 0.125\}\}$

Point number 24 of type $\{19/20, 0, 0, a, a, a, a, a, a, a, a, a, 0, 1/2 (1-9 d), 1/2 (1-9 d), d, d, d, d, d, d, d, d\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.0299073, \{a > 0.00555556, d > 0.0908683\}\}$

Point number 25 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, a, 1/20-8 a\}, \{0, 1/2 (1-8 d), 1/2 (1-8 d), d, d, d, d, d, d, d, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.018144, \{a > 0., d > 0.1\}\}$

Point number 26 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, 1/2 (1/20-7 a), 1/2 (1/20-7 a)\}, \{0, 1/2 (1-7 d), 1/2 (1-7 d), d, d, d, d, d, d, 0, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00842991, \{a > 0., d > 0.111111\}\}$

Point number 27 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, 1/3 (1/20-6 a), 1/3 (1/20-6 a), 1/3 (1/20-6 a)\}, \{0, 1/2 (1-6 d), 1/2 (1-6 d), d, d, d, d, d, 0, 0, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00240326, \{a > 0., d > 0.125\}\}$

Point number 28 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, a, a\}, \{0, 1/3 (1-8 d), 1/3 (1-8 d), 1/3 (1-8 d), d, d, d, d, d, d, d\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.0299074, \{a > 0.00625, d > 0.0908403\}\}$

Point number 29 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, a, 1/20-7 a\}, \{0, 1/3 (1-7 d), 1/3 (1-7 d), 1/3 (1-7 d), d, d, d, d, d, d, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.018144, \{a > 0., d > 0.1\}\}$

Point number 30 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, 1/2 (1/20-6 a), 1/2 (1/20-6 a)\}, \{0, 1/3 (1-6 d), 1/3 (1-6 d), 1/3 (1-6 d), d, d, d, d, d, 0, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00842991, \{a > 0., d > 0.111111\}\}$

Point number 31 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, 1/3 (1/20-5 a), 1/3 (1/20-5 a), 1/3 (1/20-5 a)\}, \{0, 1/3 (1-5 d), 1/3 (1-5 d), 1/3 (1-5 d), d, d, d, d, d, 0, 0, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00240326, \{a > 0., d > 0.125\}\}$

Point number 32 of type $\{\{19/20, 0, 0, 0, 0, a, a, a, a, a, a\}, \{0, 1/4 (1-7 d), 1/4 (1-7 d), 1/4 (1-7 d), 1/4 (1-7 d), d, d, d, d, d, d\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.0299074, \{a > 0.00714286, d > 0.0908042\}\}$

Point number 33 of type $\{\{19/20, 0, 0, 0, 0, a, a, a, a, a, 1/20-6 a\}, \{0, 1/4 (1-6 d), 1/4 (1-6 d), 1/4 (1-6 d), 1/4 (1-6 d), d, d, d, d, d, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.018144, \{a > 0., d > 0.1\}\}$

Point number 34 of type $\{\{19/20, 0, 0, 0, 0, a, a, a, a, a, 1/2 (1/20-5 a), 1/2 (1/20-5 a)\}, \{0, 1/4 (1-5 d), 1/4 (1-5 d), 1/4 (1-5 d), 1/4 (1-5 d), d, d, d, d, d, 0, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00842991, \{a > 0., d > 0.111111\}\}$

Point number 35 of type $\{\{19/20, 0, 0, 0, 0, a, a, a, a, 1/3 (1/20-4 a), 1/3 (1/20-4 a), 1/3 (1/20-4 a)\}, \{0, 1/4 (1-4 d), 1/4 (1-4 d), 1/4 (1-4 d), 1/4 (1-4 d), d, d, d, d, 0, 0, 0\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{0.00240326, \{a > 0., d > 0.125\}\}$

Point number 36 of type $\{\{19/20, a, a, a, a, a, a, a, a, a, 0\}, \{0, d, d, d, d, d, d, d, d, 19/20\}\}$ with $\{a, d\} \leq 1/20$ has maximum value equal to $\{6.94424 \cdot 10^{-10}, \{a > 0.005, d > 0.005\}\}$

Point number 37 of type $\{\{19/20, 0, a, a, a, a, a, a, a, 0\}, \{0, 1/20-9$

d,d,d,d,d,d,d,d,d,19/20}} with $\{a,d\} \leq 1/20$ has maximum value equal to $\{5.64886 \cdot 10^{-10}, \{a > 0.00555556, d > 0.00499585\}\}$

Point number 38 of type $\{\{19/20, 0, a, a, a, a, a, a, 1/20-8 a, 0\}, \{0, 1/20-8 d, d, d, d, d, d, d, 0, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{4.35544 \cdot 10^{-10}, \{a > 0.00554962, d > 0.00554962\}\}$

Point number 39 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/20-8 d), 1/2 (1/20-8 d), d, d, d, d, d, d, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{4.57467 \cdot 10^{-10}, \{a > 0.00625, d > 0.00499369\}\}$

Point number 40 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, 1/20-7 a, 0\}, \{0, 1/2 (1/20-7 d), 1/2 (1/20-7 d), d, d, d, d, d, d, 0, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{3.28261 \cdot 10^{-10}, \{a > 0.00623882, d > 0.00554751\}\}$

Point number 41 of type $\{\{19/20, 0, 0, a, a, a, a, a, 1/2 (1/20-6 a), 1/2 (1/20-6 a), 0\}, \{0, 1/2 (1/20-6 d), 1/2 (1/20-6 d), d, d, d, d, d, d, 0, 0, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{2.21008 \cdot 10^{-10}, \{a > 0.00623652, d > 0.00623652\}\}$

Point number 42 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, a, 0\}, \{0, 1/3 (1/20-7 d), 1/3 (1/20-7 d), 1/3 (1/20-7 d), d, d, d, d, d, d, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{3.82281 \cdot 10^{-10}, \{a > 0.00714286, d > 0.00499278\}\}$

Point number 43 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, 1/20-6 a, 0\}, \{0, 1/3 (1/20-6 d), 1/3 (1/20-6 d), 1/3 (1/20-6 d), d, d, d, d, d, d, 0, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{2.53193 \cdot 10^{-10}, \{a > 0.00710549, d > 0.00554809\}\}$

Point number 44 of type $\{\{19/20, 0, 0, 0, a, a, a, a, 1/2 (1/20-5 a), 1/2 (1/20-5 a), 0\}, \{0, 1/3 (1/20-5 d), 1/3 (1/20-5 d), 1/3 (1/20-5 d), d, d, d, d, d, 0, 0, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{1.45891 \cdot 10^{-10}, \{a > 0.00710109, d > 0.00624073\}\}$

Point number 45 of type $\{\{19/20, 0, 0, 0, a, a, a, 1/3 (1/20-4 a), 1/3 (1/20-4 a), 1/3 (1/20-4 a), 0\}, \{0, 1/3 (1/20-4 d), 1/3 (1/20-4 d), 1/3 (1/20-4 d), d, d, d, d, 0, 0, 0, 19/20\}\}$ with $\{a,d\} \leq 1/20$ has maximum value equal to $\{7.06007 \cdot 10^{-11}, \{a > 0.00712033, d > 0.00712033\}\}$

Point number 46 of type $\{\{19/20, 1/20, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, a, (1-a)/10, (1-a)/10, (1-a)/10, (1-a)/10, (1-a)/10, (1-a)/10, (1-a)/10, (1-a)/10\}\}$ with $\{a\} \leq 1/20$ has maximum value equal to $\{0.0299107, \{a > 0.0890411\}\}$

Point number 47 of type $\{\{19/20, 1/20, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10\}\}$ with $\{\} \leq 1/20$ has maximum value equal to 0.018144

Point number 48 of type $\{\{19/20, 1/20, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9\}\}$ with $\{\} \leq 1/20$ has maximum value equal to 0.008429910375751965

Point number 49 of type $\{\{19/20, 1/20, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8\}\}$ with $\{\} \leq 1/20$ has maximum value equal to

0.00240325927734375`

Max attained in point 1 with value 0.03094457304526749`.

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,a,a,a,a,a,a,a\},\{d,d,d,d,d,d,d,d,d,d\}\}$ with value $\{0.0309446,\{a->0.0833333,d->0.0833333\}\}$

Point number 2 of type $\{\{0,a,a,a,a,a,a,a,a,a\},\{1-11\ d,d,d,d,d,d,d,d,d,d\}\}$ with value $\{0.0310358,\{a->0.0909091,d->0.\}\}$

Point number 3 of type $\{\{0,a,a,a,a,a,a,a,a,a,1-10\ a\},\{1-10\ d,d,d,d,d,d,d,d,d,d,0\}\}$ with value $\{0.0310358,\{a->0.,d->0.0909091\}\}$

Point number 4 of type $\{\{0,0,a,a,a,a,a,a,a,a\},\{1/2\ (1-10\ d),1/2\ (1-10\ d),d,d,d,d,d,d,d,d,d,d\}\}$ with value $\{0.0221641,\{a->0.1,d->0.0785157\}\}$

Point number 5 of type $\{\{0,0,a,a,a,a,a,a,a,a,1-9\ a\},\{1/2\ (1-9\ d),1/2\ (1-9\ d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with value $\{0.0310358,\{a->0.,d->0.0909091\}\}$

Point number 6 of type $\{\{0,0,a,a,a,a,a,a,a,a,1/2\ (1-8\ a),1/2\ (1-8\ a)\},\{1/2\ (1-8\ d),1/2\ (1-8\ d),d,d,d,d,d,d,d,d,d,d,0,0\}\}$ with value $\{0.018144,\{a->0.,d->0.1\}\}$

Point number 7 of type $\{\{0,0,0,a,a,a,a,a,a,a,a\},\{1/3\ (1-9\ d),1/3\ (1-9\ d),1/3\ (1-9\ d),d,d,d,d,d,d,d,d,d,d\}\}$ with value $\{0.0188293,\{a->0.111111,d->0.0778177\}\}$

Point number 8 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,1-8\ a\},\{1/3\ (1-8\ d),1/3\ (1-8\ d),1/3\ (1-8\ d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with value $\{0.0310358,\{a->0.,d->0.0909091\}\}$

Point number 9 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,1/2\ (1-7\ a),1/2\ (1-7\ a)\},\{1/3\ (1-7\ d),1/3\ (1-7\ d),1/3\ (1-7\ d),d,d,d,d,d,d,d,d,d,d,0,0\}\}$ with value $\{0.018144,\{a->0.,d->0.1\}\}$

Point number 10 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,1/3\ (1-6\ a),1/3\ (1-6\ a),1/3\ (1-6\ a)\},\{1/3\ (1-6\ d),1/3\ (1-6\ d),1/3\ (1-6\ d),d,d,d,d,d,d,d,d,d,d,0,0,0\}\}$ with value $\{0.00842991,\{a->0.,d->0.111111\}\}$

Point number 11 of type $\{\{0,0,0,0,a,a,a,a,a,a,a,a\},\{1/4\ (1-8\ d),1/4\ (1-8\ d),1/4\ (1-8\ d),1/4\ (1-8\ d),d,d,d,d,d,d,d,d,d,d\}\}$ with value $\{0.0167487,\{a->0.125,d->0.0771487\}\}$

Point number 12 of type $\{\{0,0,0,0,a,a,a,a,a,a,a,a,1-7\ a\},\{1/4\ (1-7\ d),1/4\ (1-7\ d),1/4\ (1-7\ d),1/4\ (1-7\ d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with value $\{0.0310358,\{a->0.,d->0.0909091\}\}$

Point number 13 of type $\{\{0,0,0,0,a,a,a,a,a,a,a,a,1/2\ (1-6\ a),1/2\ (1-6\ a)\},\{1/4\ (1-6\ d),1/4\ (1-6\ d),1/4\ (1-6\ d),1/4\ (1-6\ d),d,d,d,d,d,d,d,d,d,d,0,0\}\}$ with value $\{0.018144,\{a->0.,d->0.1\}\}$

Point number 14 of type $\{\{0,0,0,0,a,a,a,a,a,a,a,a,1/3\ (1-5\ a),1/3\ (1-5\ a),1/3\ (1-5\ a)\},\{1/4\ (1-5\ d),1/4\ (1-5\ d),1/4\ (1-5\ d),1/4\ (1-5\ d),d,d,d,d,d,d,d,d,d,d,0,0,0\}\}$ with value $\{0.00842991,\{a->0.,d->0.111111\}\}$

Point number 15 of type $\{ \{0,0,0,0,a,a,a,a,1/4 (1-4 a),1/4 (1-4 a),1/4 (1-4 a),1/4 (1-4 a)\}, \{1/4 (1-4 d),1/4 (1-4 d),1/4 (1-4 d),1/4 (1-4 d),d,d,d,d,0,0,0,0\} \}$ with value $\{0.00254084,\{a->0.111111,d->0.111111\}\}$

Max attained in point 2 with value 0.031035802990593143`.

EXACT VALUE OF M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type $\{ \{19/20,a,a,a,a,a,a,a,a,a,a\}, \{19/20,d,d,d,d,d,d,d,d,d,d\} \}$ has maximum value equal to $\{2.0764*10^{-11},\{a->0.00454545,d->0.00454545\}\}$

Point number 2 of type $\{ \{19/20,0,a,a,a,a,a,a,a,a,a\}, \{19/20,1/20-10 d,d,d,d,d,d,d,d,d,d\} \}$ has maximum value equal to $\{1.86638*10^{-11},\{a->0.005,d->0.\}\}$

Point number 3 of type $\{ \{19/20,0,a,a,a,a,a,a,a,a,1/20-9 a\}, \{19/20,1/20-9 d,d,d,d,d,d,d,d,d,0\} \}$ has maximum value equal to $\{1.86638*10^{-11},\{a->0.,d->0.005\}\}$

Point number 4 of type $\{ \{19/20,0,0,a,a,a,a,a,a,a,a\}, \{19/20,1/2 (1/20-9 d),1/2 (1/20-9 d),d,d,d,d,d,d,d,d,d\} \}$ has maximum value equal to $\{1.52223*10^{-11},\{a->0.00555556,d->0.00424741\}\}$

Point number 5 of type $\{ \{19/20,0,0,a,a,a,a,a,a,a,1/20-8 a\}, \{19/20,1/2 (1/20-8 d),1/2 (1/20-8 d),d,d,d,d,d,d,d,d,0\} \}$ has maximum value equal to $\{1.86638*10^{-11},\{a->0.,d->0.005\}\}$

Point number 6 of type $\{ \{19/20,0,0,a,a,a,a,a,a,a,1/2 (1/20-7 a),1/2 (1/20-7 a)\}, \{19/20,1/2 (1/20-7 d),1/2 (1/20-7 d),d,d,d,d,d,d,d,0,0\} \}$ has maximum value equal to $\{1.15911*10^{-11},\{a->0.,d->0.00555556\}\}$

Point number 7 of type $\{ \{19/20,0,0,0,a,a,a,a,a,a,a\}, \{19/20,1/3 (1/20-8 d),1/3 (1/20-8 d),1/3 (1/20-8 d),d,d,d,d,d,d,d,d\} \}$ has maximum value equal to $\{1.29357*10^{-11},\{a->0.00625,d->0.00418966\}\}$

Point number 8 of type $\{ \{19/20,0,0,0,a,a,a,a,a,a,a,1/20-7 a\}, \{19/20,1/3 (1/20-7 d),1/3 (1/20-7 d),1/3 (1/20-7 d),d,d,d,d,d,d,d,0\} \}$ has maximum value equal to $\{1.86638*10^{-11},\{a->0.,d->0.005\}\}$

Point number 9 of type $\{ \{19/20,0,0,0,a,a,a,a,a,a,1/2 (1/20-6 a),1/2 (1/20-6 a)\}, \{19/20,1/3 (1/20-6 d),1/3 (1/20-6 d),1/3 (1/20-6 d),d,d,d,d,d,d,d,0,0\} \}$ has maximum value equal to $\{1.15911*10^{-11},\{a->0.,d->0.00555556\}\}$

Point number 10 of type $\{ \{19/20,0,0,0,a,a,a,a,a,1/3 (1/20-5 a),1/3 (1/20-5 a),1/3 (1/20-5 a)\}, \{19/20,1/3 (1/20-5 d),1/3 (1/20-5 d),1/3 (1/20-5 d),d,d,d,d,d,0,0,0\} \}$ has maximum value equal to $\{5.80162*10^{-12},\{a->0.,d->0.00625\}\}$

Point number 11 of type $\{ \{19/20,0,0,0,0,a,a,a,a,a,a\}, \{19/20,1/4 (1/20-7 d),1/4 (1/20-7 d),1/4 (1/20-7 d),1/4 (1/20-7 d),d,d,d,d,d,d,d\} \}$ has maximum value equal to

{1.13792*10⁻¹¹, {a->0.00714286, d->0.00414328}}

Point number 12 of type {{19/20,0,0,0,0,a,a,a,a,1/20-6 a},{19/20,1/4 (1/20-6 d),1/4 (1/20-6 d),1/4 (1/20-6 d),1/4 (1/20-6 d),d,d,d,d,d,0}} has maximum value equal to {1.86638*10⁻¹¹, {a->0., d->0.005}}

Point number 13 of type {{19/20,0,0,0,0,a,a,a,a,1/2 (1/20-5 a),1/2 (1/20-5 a)}, {19/20,1/4 (1/20-5 d),1/4 (1/20-5 d),1/4 (1/20-5 d),1/4 (1/20-5 d),d,d,d,d,d,0,0}} has maximum value equal to {1.15911*10⁻¹¹, {a->0., d->0.00555556}}

Point number 14 of type {{19/20,0,0,0,0,a,a,a,a,1/3 (1/20-4 a),1/3 (1/20-4 a),1/3 (1/20-4 a)}, {19/20,1/4 (1/20-4 d),1/4 (1/20-4 d),1/4 (1/20-4 d),1/4 (1/20-4 d),d,d,d,d,d,0,0,0}} has maximum value equal to {5.80162*10⁻¹², {a->0., d->0.00625}}

Point number 15 of type {{19/20,0,0,0,0,a,a,a,1/4 (1/20-3 a),1/4 (1/20-3 a),1/4 (1/20-3 a),1/4 (1/20-3 a)}, {19/20,1/4 (1/20-3 d),1/4 (1/20-3 d),1/4 (1/20-3 d),1/4 (1/20-3 d),d,d,d,0,0,0,0}} has maximum value equal to {2.17733*10⁻¹², {a->0.00625, d->0.00625}}

Point number 16 of type {{19/20,1/20,0,0,0,0,0,0,0,0,0,0}, {19/20,a,1/10 (1/20-a),1/10 (1/20-a),1/10 (1/20-a),1/10 (1/20-a),1/10 (1/20-a),1/10 (1/20-a),1/10 (1/20-a),1/10 (1/20-a),1/10 (1/20-a)}} has maximum value equal to {1.86638*10⁻¹¹, {a->0.}}

Point number 17 of type {{19/20,1/20,0,0,0,0,0,0,0,0,0,0}, {19/20,0,1/200,1/200,1/200,1/200,1/200,1/200,1/200,1/200,1/200,1/200}} has maximum value equal to 1.866375*10⁻¹¹

Point number 18 of type {{19/20,1/20,0,0,0,0,0,0,0,0,0,0}, {19/20,0,0,1/180,1/180,1/180,1/180,1/180,1/180,1/180,1/180,1/180}} has maximum value equal to 1.1591126766658952*10⁻¹¹

Point number 19 of type {{19/20,1/20,0,0,0,0,0,0,0,0,0,0}, {19/20,0,0,0,1/160,1/160,1/160,1/160,1/160,1/160,1/160,1/160}} has maximum value equal to 5.801618099212647*10⁻¹²

Point number 20 of type {{19/20,1/20,0,0,0,0,0,0,0,0,0,0}, {19/20,0,0,0,0,1/140,1/140,1/140,1/140,1/140,1/140,1/140}} has maximum value equal to 1.816845022057136*10⁻¹²

Max attained in point 1 with value 2.0764010239445128*10⁻¹¹.

EXACT VALUE OF M_4 (Unbalanced-Unbalanced distributions on different coordinates)

Point number 1 of type {{19/20,a,a,a,a,a,a,a,a,0},{0,d,d,d,d,d,d,d,d,19/20}} has maximum value equal to {6.94424*10⁻¹⁰, {a->0.005, d->0.005}}

Point number 2 of type {{19/20,0,a,a,a,a,a,a,a,0},{0,1/20-9 d,d,d,d,d,d,d,d,19/20}} has maximum value equal to {5.64886*10⁻¹⁰, {a-

$>0.00555556, d \rightarrow 0.00499585\}$

Point number 3 of type $\{\{19/20, 0, a, a, a, a, a, a, 1/20-8 a, 0\}, \{0, 1/20-8 d, d, d, d, d, d, d, d, 0, 19/20\}\}$ has maximum value equal to $\{4.35544 \cdot 10^{-10}, \{a \rightarrow 0.00554962, d \rightarrow 0.00554962\}\}$

Point number 4 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/20-8 d), 1/2 (1/20-8 d), d, d, d, d, d, d, d, 19/20\}\}$ has maximum value equal to $\{4.57467 \cdot 10^{-10}, \{a \rightarrow 0.00625, d \rightarrow 0.00499369\}\}$

Point number 5 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, 1/20-7 a, 0\}, \{0, 1/2 (1/20-7 d), 1/2 (1/20-7 d), d, d, d, d, d, d, d, 0, 19/20\}\}$ has maximum value equal to $\{3.28261 \cdot 10^{-10}, \{a \rightarrow 0.00623882, d \rightarrow 0.00554751\}\}$

Point number 6 of type $\{\{19/20, 0, 0, a, a, a, a, a, a, 1/2 (1/20-6 a), 1/2 (1/20-6 a), 0\}, \{0, 1/2 (1/20-6 d), 1/2 (1/20-6 d), d, d, d, d, d, d, d, 0, 0, 19/20\}\}$ has maximum value equal to $\{2.21008 \cdot 10^{-10}, \{a \rightarrow 0.00623652, d \rightarrow 0.00623652\}\}$

Point number 7 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, a, 0\}, \{0, 1/3 (1/20-7 d), 1/3 (1/20-7 d), 1/3 (1/20-7 d), d, d, d, d, d, d, d, 19/20\}\}$ has maximum value equal to $\{3.82281 \cdot 10^{-10}, \{a \rightarrow 0.00714286, d \rightarrow 0.00499278\}\}$

Point number 8 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, a, 1/20-6 a, 0\}, \{0, 1/3 (1/20-6 d), 1/3 (1/20-6 d), 1/3 (1/20-6 d), d, d, d, d, d, d, d, 0, 19/20\}\}$ has maximum value equal to $\{2.53193 \cdot 10^{-10}, \{a \rightarrow 0.00710549, d \rightarrow 0.00554809\}\}$

Point number 9 of type $\{\{19/20, 0, 0, 0, a, a, a, a, a, a, 1/2 (1/20-5 a), 1/2 (1/20-5 a), 0\}, \{0, 1/3 (1/20-5 d), 1/3 (1/20-5 d), 1/3 (1/20-5 d), d, d, d, d, d, d, d, 0, 0, 19/20\}\}$ has maximum value equal to $\{1.45891 \cdot 10^{-10}, \{a \rightarrow 0.00710109, d \rightarrow 0.00624073\}\}$

Point number 10 of type $\{\{19/20, 0, 0, 0, a, a, a, a, 1/3 (1/20-4 a), 1/3 (1/20-4 a), 1/3 (1/20-4 a), 0\}, \{0, 1/3 (1/20-4 d), 1/3 (1/20-4 d), 1/3 (1/20-4 d), d, d, d, d, d, d, d, 0, 0, 19/20\}\}$ has maximum value equal to $\{7.06007 \cdot 10^{-11}, \{a \rightarrow 0.00712033, d \rightarrow 0.00712033\}\}$

Point number 11 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1/200, 1/200, 1/200, 1/200, 1/200, 1/200, 1/200, 1/200, 1/200, 1/200, 19/20\}\}$ has maximum value equal to $3.5980875 \cdot 10^{-10}$

Point number 12 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/180, 1/180, 1/180, 1/180, 1/180, 1/180, 1/180, 1/180, 1/180, 1/180, 19/20\}\}$ has maximum value equal to $2.2556596122617562 \cdot 10^{-10}$

Point number 13 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1/160, 1/160, 1/160, 1/160, 1/160, 1/160, 1/160, 1/160, 1/160, 19/20\}\}$ has maximum value equal to $1.1424869298934936 \cdot 10^{-10}$

Point number 14 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1/140, 1/140, 1/140, 1/140, 1/140, 1/140, 1/140, 19/20\}\}$ has maximum value equal to $3.6336900441142724 \cdot 10^{-11}$

Max attained in point 1 with value $6.944238 \cdot 10^{-10}$.

UPPER BOUND ON THE RATE

The bound on the rate for $b=12$, $k=10$ and $\epsilon=1/20$ is 0.029760522473128086 and $\eta_0=0.9970691087919844$