

Authors: S. Della Fiore, S. Costa, M. Dalai

Title: Mathematica Output for the upper bound when $B=K=7$ and $\epsilon=9/100$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,a,a,a,a\},\{d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0856786,\{a>0.142857,d>0.142857\}\}$

Point number 2 of type $\{\{0,a,a,a,a,a,a\},\{1-6\,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0856759,\{a>0.166667,d>0.015\}\}$

Point number 3 of type $\{\{0,a,a,a,a,a,1-5\,a\},\{1-5\,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0856679,\{a>0.018,d>0.166528\}\}$

Point number 4 of type $\{\{0,0,a,a,a,a,a\},\{1/2\,(1-5\,d),1/2\,(1-5\,d),d,d,d,d,d\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0584977,\{a>0.2,d>0.120882\}\}$

Point number 5 of type $\{\{0,0,a,a,a,a,1-4\,a\},\{1/2\,(1-4\,d),1/2\,(1-4\,d),d,d,d,d,0\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0856589,\{a>0.0225,d>0.166325\}\}$

Point number 6 of type $\{\{0,0,a,a,a,1/2\,(1-3\,a),1/2\,(1-3\,a)\},\{1/2\,(1-3\,d),1/2\,(1-3\,d),d,d,d,0,0\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0352126,\{a>0.03,d>0.199693\}\}$

Point number 7 of type $\{\{91/100,a,a,a,a,a,a\},\{0,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0856759,\{a>0.015,d>0.166667\}\}$

Point number 8 of type $\{\{91/100,a,a,a,a,a,9/100-5\,a\},\{0,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0384,\{a>0.,d>0.2\}\}$

Point number 9 of type $\{\{91/100,0,a,a,a,a,a\},\{0,1-5\,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0856679,\{a>0.018,d>0.166528\}\}$

Point number 10 of type $\{\{91/100,0,a,a,a,a,9/100-4\,a\},\{0,1-4\,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0384,\{a>0.,d>0.2\}\}$

Point number 11 of type $\{\{91/100,0,0,a,a,a,a\},\{0,1/2\,(1-4\,d),1/2\,(1-4\,d),d,d,d,d\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0856589,\{a>0.0225,d>0.166325\}\}$

Point number 12 of type $\{\{91/100,0,0,a,a,a,9/100-3\,a\},\{0,1/2\,(1-3\,d),1/2\,(1-3\,d),d,d,d,0\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0384,\{a>0.,d>0.2\}\}$

Point number 13 of type $\{\{91/100,a,a,a,a,a,0\},\{0,d,d,d,d,d,91/100\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.000106793,\{a>0.018,d>0.018\}\}$

Point number 14 of type $\{\{91/100,0,a,a,a,a,0\},\{0,9/100-4\,d,d,d,d,d,91/100\}\}$ with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0000793753,\{a>0.0225,d>0.0179146\}\}$

Point number 15 of type $\{\{91/100,0,a,a,a,9/100-3\,a,0\},\{0,9/100-3\,d,d,d,d,0,91/100\}\}$

with $\{a,d\} \leq 9/100$ has maximum value equal to $\{0.0000522068, \{a \rightarrow 0.0223627, d \rightarrow 0.0223627\}\}$

Point number 16 of type $\{\{91/100, 9/100, 0, 0, 0, 0, 0\}, \{0, a, (1-a)/5, (1-a)/5, (1-a)/5, (1-a)/5\}\}$ with $\{a\} \leq 9/100$ has maximum value equal to $\{0.0856626, \{a \rightarrow 0.163218\}\}$

Point number 17 of type $\{\{91/100, 9/100, 0, 0, 0, 0, 0\}, \{0, 0, 1/5, 1/5, 1/5, 1/5, 1/5\}\}$ with $\{\} \leq 9/100$ has maximum value equal to 0.0384

Max attained in point 1 with value 0.08567858630332599 .

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a, a, a, a, a, a, a\}, \{d, d, d, d, d, d, d\}\}$ with value $\{0.0856786, \{a \rightarrow 0.142857, d \rightarrow 0.142857\}\}$

Point number 2 of type $\{\{0, a, a, a, a, a, a\}, \{1-6 d, d, d, d, d, d, d\}\}$ with value $\{0.0925926, \{a \rightarrow 0.166667, d \rightarrow 0.\}\}$

Point number 3 of type $\{\{0, a, a, a, a, a, 1-5 a\}, \{1-5 d, d, d, d, d, d, 0\}\}$ with value $\{0.0925926, \{a \rightarrow 0., d \rightarrow 0.166667\}\}$

Point number 4 of type $\{\{0, 0, a, a, a, a, a\}, \{1/2 (1-5 d), 1/2 (1-5 d), d, d, d, d, d\}\}$ with value $\{0.0584977, \{a \rightarrow 0.2, d \rightarrow 0.120882\}\}$

Point number 5 of type $\{\{0, 0, a, a, a, a, 1-4 a\}, \{1/2 (1-4 d), 1/2 (1-4 d), d, d, d, d, 0\}\}$ with value $\{0.0925926, \{a \rightarrow 0., d \rightarrow 0.166667\}\}$

Point number 6 of type $\{\{0, 0, a, a, a, 1/2 (1-3 a), 1/2 (1-3 a)\}, \{1/2 (1-3 d), 1/2 (1-3 d), d, d, d, 0, 0\}\}$ with value $\{0.0384, \{a \rightarrow 0., d \rightarrow 0.2\}\}$

Max attained in point 2 with value 0.09259259259259259 .

EXACT VALUE OF M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type $\{\{91/100, a, a, a, a, a, a\}, \{91/100, d, d, d, d, d, d\}\}$ has maximum value equal to $\{5.98691 \cdot 10^{-6}, \{a \rightarrow 0.015, d \rightarrow 0.015\}\}$

Point number 2 of type $\{\{91/100, 0, a, a, a, a, a\}, \{91/100, 9/100-5 d, d, d, d, d, d\}\}$ has maximum value equal to $\{5.38527 \cdot 10^{-6}, \{a \rightarrow 0.018, d \rightarrow 0.\}\}$

Point number 3 of type $\{\{91/100, 0, a, a, a, a, 9/100-4 a\}, \{91/100, 9/100-4 d, d, d, d, d, 0\}\}$ has maximum value equal to $\{5.38527 \cdot 10^{-6}, \{a \rightarrow 0., d \rightarrow 0.018\}\}$

Point number 4 of type $\{\{91/100, 0, 0, a, a, a, a\}, \{91/100, 1/2 (9/100-4 d), 1/2 (9/100-4 d), d, d, d, d\}\}$ has maximum value equal to $\{4.19734 \cdot 10^{-6}, \{a \rightarrow 0.0225, d \rightarrow 0.0119564\}\}$

Point number 5 of type $\{\{91/100, 0, 0, a, a, a, 9/100-3 a\}, \{91/100, 1/2 (9/100-3 d), 1/2 (9/100-3 d), d, d, d, 0\}\}$ has maximum value equal to $\{5.38527 \cdot 10^{-6}, \{a \rightarrow 0., d \rightarrow 0.018\}\}$

Point number 6 of type $\{\{91/100,0,0,a,a,1/2(9/100-2a),1/2(9/100-2a)\},\{91/100,1/2(9/100-2d),1/2(9/100-2d),d,d,0,0\}\}$ has maximum value equal to $\{2.7856 \cdot 10^{-6}, \{a > 0.018, d > 0.018\}\}$

Point number 7 of type $\{\{91/100,9/100,0,0,0,0,0\},\{91/100,a,1/5(9/100-a),1/5(9/100-a),1/5(9/100-a),1/5(9/100-a)\}\}$ has maximum value equal to $\{5.38527 \cdot 10^{-6}, \{a > 0.\}\}$

Point number 8 of type $\{\{91/100,9/100,0,0,0,0,0\},\{91/100,0,9/500,9/500,9/500,9/500\}\}$ has maximum value equal to $5.3852688 \cdot 10^{-6}$

Point number 9 of type $\{\{91/100,9/100,0,0,0,0,0\},\{91/100,0,0,9/400,9/400,9/400,9/400\}\}$ has maximum value equal to $2.51880890625 \cdot 10^{-6}$

Max attained in point 1 with value $5.9869125 \cdot 10^{-6}$.

EXACT VALUE OF M_4 (Unbalanced-Unbalanced distributions on different coordinates)

Point number 1 of type $\{\{91/100,a,a,a,a,a,0\},\{0,d,d,d,d,91/100\}\}$ has maximum value equal to $\{0.000106793, \{a > 0.018, d > 0.018\}\}$

Point number 2 of type $\{\{91/100,0,a,a,a,a,0\},\{0,9/100-4d,d,d,d,d,91/100\}\}$ has maximum value equal to $\{0.0000793753, \{a > 0.0225, d > 0.0179146\}\}$

Point number 3 of type $\{\{91/100,0,a,a,a,9/100-3a,0\},\{0,9/100-3d,d,d,d,0,91/100\}\}$ has maximum value equal to $\{0.0000522068, \{a > 0.0223627, d > 0.0223627\}\}$

Point number 4 of type $\{\{1,0,0,0,0,0,0\},\{0,9/500,9/500,9/500,9/500,9/500,91/100\}\}$ has maximum value equal to 0.00005754364416

Point number 5 of type $\{\{1,0,0,0,0,0,0\},\{0,0,9/400,9/400,9/400,9/400,91/100\}\}$ has maximum value equal to 0.000027986765625

Max attained in point 1 with value 0.0001067928406272 .

UPPER BOUND ON THE RATE

The bound on the rate for $b=7$, $k=7$ and $\epsilon=9/100$ is 0.040897467564271935 and $\eta_0=0.9304525328351868$