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Title: Mathematica Output for the upper bound when $B=6$, $K=5$ and $\epsilon=1/10$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,a,a,a\},\{d,d,d,d,d,d\}\}$ with $\{a,d\} \geq 1/10$ has maximum equal to $\{0.555556,\{a \rightarrow 0.166667,d \rightarrow 0.166667\}\}$

Point number 2 of type $\{\{1/10,a,a,a,a,a\},\{c,d,d,d,d,d\}\}$ with $\{a,c,d\} \geq 1/10$ has maximum equal to $\{0.555625,\{a \rightarrow 0.18,c \rightarrow 0.234204,d \rightarrow 0.153159\}\}$

Point number 3 of type $\{\{1/10,a,a,a,a,b\},\{c,d,d,d,d,1/10\}\}$ with $\{a,b,c,d\} \geq 1/10$ has maximum equal to $\{0.555556,\{a \rightarrow 0.166667,b \rightarrow 0.233333,c \rightarrow 0.233333,d \rightarrow 0.166667\}\}$

Point number 4 of type $\{\{1/10,1/10,a,a,a,a\},\{c,c,d,d,d,d\}\}$ with $\{a,c,d\} \geq 1/10$ has maximum equal to $\{0.555042,\{a \rightarrow 0.2,c \rightarrow 0.230819,d \rightarrow 0.13459\}\}$

Point number 5 of type $\{\{1/10,1/10,a,a,a,b\},\{c,c,d,d,d,1/10\}\}$ with $\{a,b,c,d\} \geq 1/10$ has maximum equal to $\{0.55498,\{a \rightarrow 0.18857,b \rightarrow 0.234291,c \rightarrow 0.230737,d \rightarrow 0.146175\}\}$

Point number 6 of type $\{\{1/10,1/10,a,a,b,b\},\{c,c,d,d,1/10,1/10\}\}$ with $\{a,b,c,d\} \geq 1/10$ has maximum equal to $\{0.554624,\{a \rightarrow 0.167823,b \rightarrow 0.232177,c \rightarrow 0.232177,d \rightarrow 0.167823\}\}$

Point number 7 of type $\{\{1/10,1/10,1/10,a,a,a\},\{c,c,c,d,d,d\}\}$ with $\{a,c,d\} \geq 1/10$ has maximum equal to $\{0.552974,\{a \rightarrow 0.233333,c \rightarrow 0.227164,d \rightarrow 0.10617\}\}$

Point number 8 of type $\{\{1/10,1/10,1/10,a,a,b\},\{c,c,c,d,d,1/10\}\}$ with $\{a,b,c,d\} \geq 1/10$ has maximum equal to $\{0.552974,\{a \rightarrow 0.230217,b \rightarrow 0.239567,c \rightarrow 0.227145,d \rightarrow 0.109283\}\}$

Point number 9 of type $\{\{1/10,1/10,1/10,a,b,b\},\{c,c,c,d,1/10,1/10\}\}$ with $\{a,b,c,d\} \geq 1/10$ has maximum equal to $\{0.55297,\{a \rightarrow 0.221444,b \rightarrow 0.239278,c \rightarrow 0.227305,d \rightarrow 0.118085\}\}$

Point number 10 of type $\{\{1/10,1/10,1/10,b,b,b\},\{c,c,c,1/10,1/10,1/10\}\}$ with $\{b,c\} \geq 1/10$ has maximum equal to $\{0.552711,\{b \rightarrow 0.233333,c \rightarrow 0.233333\}\}$

Point number 11 of type $\{\{1/10,1/10,1/10,1/10,a,a\},\{c,c,c,c,d,d\}\}$ with $\{a,c,d\} \geq 1/10$ has maximum equal to $\{0.54,\{a \rightarrow 0.3,c \rightarrow 0.2,d \rightarrow 0.1\}\}$

Point number 12 of type $\{\{1/10,1/10,1/10,1/10,a,b\},\{c,c,c,c,d,1/10\}\}$ with $\{a,b,c,d\} \geq 1/10$ has maximum equal to $\{0.54,\{a \rightarrow 0.3,b \rightarrow 0.3,c \rightarrow 0.2,d \rightarrow 0.1\}\}$

Point number 13 of type $\{\{1/10,1/10,1/10,1/10,b,b\},\{c,c,c,c,1/10,1/10\}\}$ with $\{b,c\} \geq 1/10$ has maximum equal to $\{0.54,\{b \rightarrow 0.3,c \rightarrow 0.2\}\}$

Point number 14 of type $\{\{1/10, 1/10, 1/10, 1/10, b, b\}, \{c, c, c, 1/10, 1/10, 1/10\}\}$ with $\{b, c\} \geq 1/10$ has maximum equal to $\{0.523556, \{b \rightarrow 0.3, c \rightarrow 0.233333\}\}$

Point number 15 of type $\{\{1/10, 1/10, 1/10, 1/10, b, b\}, \{c, c, 1/10, 1/10, 1/10, 1/10\}\}$ with $\{b, c\} \geq 1/10$ has maximum equal to $\{0.4944, \{b \rightarrow 0.3, c \rightarrow 0.3\}\}$

Point number 16 of type $\{\{1/10, 1/10, 1/10, 1/10, 1/10, a\}, \{c, c, c, c, d\}\}$ with $\{a, c, d\} \geq 1/10$ has maximum equal to $\{0.492864, \{a \rightarrow 0.5, c \rightarrow 0.18, d \rightarrow 0.1\}\}$

Point number 17 of type $\{\{1/10, 1/10, 1/10, 1/10, 1/10, b\}, \{c, c, c, c, c, 1/10\}\}$ with $\{b, c\} \geq 1/10$ has maximum equal to $\{0.492864, \{b \rightarrow 0.5, c \rightarrow 0.18\}\}$

Point number 18 of type $\{\{1/10, 1/10, 1/10, 1/10, 1/10, b\}, \{c, c, c, c, 1/10, 1/10\}\}$ with $\{b, c\} \geq 1/10$ has maximum equal to $\{0.4824, \{b \rightarrow 0.5, c \rightarrow 0.2\}\}$

Point number 19 of type $\{\{1/10, 1/10, 1/10, 1/10, 1/10, b\}, \{c, c, c, 1/10, 1/10, 1/10\}\}$ with $\{b, c\} \geq 1/10$ has maximum equal to $\{0.465956, \{b \rightarrow 0.5, c \rightarrow 0.233333\}\}$

Point number 20 of type $\{\{1/10, 1/10, 1/10, 1/10, 1/10, b\}, \{c, c, 1/10, 1/10, 1/10, 1/10\}\}$ with $\{b, c\} \geq 1/10$ has maximum equal to $\{0.4368, \{b \rightarrow 0.5, c \rightarrow 0.3\}\}$

Point number 21 of type $\{\{1/10, 1/10, 1/10, 1/10, 1/10, b\}, \{c, 1/10, 1/10, 1/10, 1/10, 1/10\}\}$ with $\{b, c\} \geq 1/10$ has maximum equal to $\{0.3792, \{b \rightarrow 0.5, c \rightarrow 0.5\}\}$

Max attained in point 2 with value 0.5556247138505845`.

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a, a, a, a, a, b\}, \{c, c, c, c, c, 1/10\}\}$ with conditions $\{a \leq 1/10, b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.492864, \{a \rightarrow 0.1, b \rightarrow 0.5, c \rightarrow 0.18\}\}$

Point number 2 of type $\{\{a, a, a, a, b, b\}, \{c, c, c, c, 1/10, 1/10\}\}$ with conditions $\{a \leq 1/10, b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.54, \{a \rightarrow 0.1, b \rightarrow 0.3, c \rightarrow 0.2\}\}$

Point number 3 of type $\{\{a, a, a, b, b, b\}, \{c, c, c, 1/10, 1/10, 1/10\}\}$ with conditions $\{a \leq 1/10, b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.552711, \{a \rightarrow 0.1, b \rightarrow 0.233333, c \rightarrow 0.233333\}\}$

Point number 4 of type $\{\{a, a, b, b, b, b\}, \{c, c, 1/10, 1/10, 1/10, 1/10\}\}$ with conditions $\{a \leq 1/10, b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.548141, \{a \rightarrow 0.0509506, b \rightarrow 0.224525, c \rightarrow 0.3\}\}$

Point number 5 of type $\{\{a, b, b, b, b, b\}, \{c, 1/10, 1/10, 1/10, 1/10, 1/10\}\}$ with conditions $\{a \leq 1/10, b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.54, \{a \rightarrow 0., b \rightarrow 0.2, c \rightarrow 0.5\}\}$

Point number 6 of type $\{\{1/10, a, a, a, a, a\}, \{c, d, d, d, d, d\}\}$ with conditions $\{a \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.555625, \{a \rightarrow 0.18, c \rightarrow 0.234204, d \rightarrow 0.153159\}\}$

Point number 7 of type $\{\{1/10, a, a, a, a, b\}, \{c, d, d, d, d, 1/10\}\}$ with conditions

$\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.555556, \{a > 0.166667, b > 0.233333, c > 0.233333, d > 0.166667\}\}$

Point number 8 of type $\{\{1/10, a, a, a, b, b\}, \{c, d, d, d, 1/10, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.55498, \{a > 0.146175, b > 0.230737, c > 0.234291, d > 0.18857\}\}$

Point number 9 of type $\{\{1/10, a, a, b, b, b\}, \{c, d, d, 1/10, 1/10, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.552974, \{a > 0.109283, b > 0.227145, c > 0.239567, d > 0.230217\}\}$

Point number 10 of type $\{\{1/10, a, b, b, b, b\}, \{c, d, 1/10, 1/10, 1/10, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.551438, \{a > 0., b > 0.225, c > 0.232639, d > 0.367361\}\}$

Point number 11 of type $\{\{1/10, b, b, b, b, b\}, \{c, 1/10, 1/10, 1/10, 1/10, 1/10\}\}$ with conditions $\{b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.492864, \{b > 0.18, c > 0.5\}\}$

Point number 12 of type $\{\{0, a, a, a, a, a\}, \{c, d, d, d, d, d\}\}$ with conditions $\{a \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.558469, \{a > 0.2, c > 0.348915, d > 0.130217\}\}$

Point number 13 of type $\{\{0, a, a, a, a, b\}, \{c, d, d, d, d, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.558138, \{a > 0.193524, b > 0.225905, c > 0.350051, d > 0.137487\}\}$

Point number 14 of type $\{\{0, a, a, a, b, b\}, \{c, d, d, d, 1/10, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.557521, \{a > 0.183507, b > 0.224739, c > 0.352843, d > 0.149052\}\}$

Point number 15 of type $\{\{0, a, a, b, b, b\}, \{c, d, d, 1/10, 1/10, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.556157, \{a > 0.166335, b > 0.222444, c > 0.360574, d > 0.169713\}\}$

Point number 16 of type $\{\{0, a, b, b, b, b\}, \{c, d, 1/10, 1/10, 1/10, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.552319, \{a > 0.13184, b > 0.21704, c > 0.386723, d > 0.213277\}\}$

Point number 17 of type $\{\{0, b, b, b, b, b\}, \{c, 1/10, 1/10, 1/10, 1/10, 1/10\}\}$ with conditions $\{b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.54, \{b > 0.2, c > 0.5\}\}$

Point number 18 of type $\{\{0, 0, a, a, a, a\}, \{c, c, d, d, d, d\}\}$ with conditions $\{a \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.5385, \{a > 0.25, c > 0.3, d > 0.1\}\}$

Point number 19 of type $\{\{0, 0, a, a, a, b\}, \{c, c, d, d, d, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.5385, \{a > 0.25, b > 0.25, c > 0.3, d > 0.1\}\}$

Point number 20 of type $\{\{0, 0, a, a, b, b\}, \{c, c, d, d, 1/10, 1/10\}\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.5385, \{a > 0.25, b > 0.25, c > 0.3, d > 0.1\}\}$

$\{a > 0.3, d > 0.1\}$

Point number 21 of type $\{0, 0, a, b, b, b\}, \{c, c, d, 1/10, 1/10, 1/10\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.5385, \{a > 0.25, b > 0.25, c > 0.3, d > 0.1\}\}$

Point number 22 of type $\{0, 0, b, b, b, b\}, \{c, c, 1/10, 1/10, 1/10, 1/10\}$ with conditions $\{b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.5385, \{b > 0.25, c > 0.3\}\}$

Point number 23 of type $\{0, 0, b, b, b, b\}, \{c, 1/10, 1/10, 1/10, 1/10, 1/10\}$ with conditions $\{b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.4665, \{b > 0.25, c > 0.5\}\}$

Point number 24 of type $\{0, 0, 0, a, a, a\}, \{c, c, c, d, d, d\}$ with conditions $\{a \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.469778, \{a > 0.333333, c > 0.233333, d > 0.1\}\}$

Point number 25 of type $\{0, 0, 0, a, a, b\}, \{c, c, c, d, d, 1/10\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.469778, \{a > 0.333333, b > 0.333333, c > 0.233333, d > 0.1\}\}$

Point number 26 of type $\{0, 0, 0, a, b, b\}, \{c, c, c, d, 1/10, 1/10\}$ with conditions $\{a \leq 1, b \leq 1, c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.469778, \{a > 0.333333, b > 0.333333, c > 0.233333, d > 0.1\}\}$

Point number 27 of type $\{0, 0, 0, b, b, b\}, \{c, c, c, 1/10, 1/10, 1/10\}$ with conditions $\{b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.469778, \{b > 0.333333, c > 0.233333\}\}$

Point number 28 of type $\{0, 0, 0, b, b, b\}, \{c, c, 1/10, 1/10, 1/10, 1/10\}$ with conditions $\{b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.431556, \{b > 0.333333, c > 0.3\}\}$

Point number 29 of type $\{0, 0, 0, b, b, b\}, \{c, 1/10, 1/10, 1/10, 1/10, 1/10\}$ with conditions $\{b \leq 1, c \geq 1/10\}$ has maximum equal to $\{0.359556, \{b > 0.333333, c > 0.5\}\}$

Point number 30 of type $\{0, 0, 0, 0, 0, 1\}, \{c, c, c, c, c, d\}$ with conditions $\{c \geq 1/10, d \geq 1/10\}$ has maximum equal to $\{0.34992, \{c > 0.18, d > 0.1\}\}$

Point number 31 of type $\{0, 0, 0, 0, 0, 1\}, \{c, c, c, c, c, 1/10\}$ with conditions $\{c \geq 1/10\}$ has maximum equal to $\{0.34992, \{c > 0.18\}\}$

Point number 32 of type $\{0, 0, 0, 0, 0, 1\}, \{c, c, c, c, 1/10, 1/10\}$ with conditions $\{c \geq 1/10\}$ has maximum equal to $\{0.336, \{c > 0.2\}\}$

Point number 33 of type $\{0, 0, 0, 0, 0, 1\}, \{c, c, c, 1/10, 1/10, 1/10\}$ with conditions $\{c \geq 1/10\}$ has maximum equal to $\{0.314222, \{c > 0.233333\}\}$

Point number 34 of type $\{0, 0, 0, 0, 0, 1\}, \{c, c, 1/10, 1/10, 1/10, 1/10\}$ with conditions $\{c \geq 1/10\}$ has maximum equal to $\{0.276, \{c > 0.3\}\}$

Point number 35 of type $\{0, 0, 0, 0, 0, 1\}, \{c, 1/10, 1/10, 1/10, 1/10, 1/10\}$ with conditions $\{c \geq 1/10\}$ has maximum equal to $\{0.204, \{c > 0.5\}\}$

Max attained in point 12 with value 0.5584691723152507`.

UPPER BOUND ON M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type $\{s_1, (1-s_1)/5, (1-s_1)/5, (1-s_1)/5, (1-s_1)/5, (1-s_1)/5, s_2, (1-s_2)/5, (1-s_2)/5, (1-s_2)/5, (1-s_2)/5, (1-s_2)/5\}$ with conditions $\{s_1 \leq 1/10, s_2 \leq 1/10\}$ has maximum in $\{0.531878, \{s_1 > 0.1, s_2 > 0.1\}\}$

Point number 2 of type $\{s_1, a, a, a, a, b, s_2, (1-s_2)/4, (1-s_2)/4, (1-s_2)/4, (1-s_2)/4, 0\}$ with conditions $\{s_1 \leq 1/10, s_2 \leq 1/10\}$ has maximum in $\{0.535106, \{s_1 > 0.1, s_2 > 0.1, a > 0.130768, b > 0.376927\}\}$

Point number 3 of type $\{s_1, a, a, a, b, b, s_2, (1-s_2)/3, (1-s_2)/3, (1-s_2)/3, 0, 0\}$ with conditions $\{s_1 \leq 1/10, s_2 \leq 1/10\}$ has maximum in $\{0.503792, \{s_1 > 0.1, s_2 > 0.1, a > 0.0900202, b > 0.31497\}\}$

Point number 4.1 of type $\{a, 0, a, a, a, b, d, c, d, d, d, 0\}$ with conditions $\{a \leq 1/10, d \leq 1/10\}$ has maximum in $\{0.3792, \{a > 0.1, b > 0.6, c > 0.6, d > 0.1\}\}$

Point number 4.2.1 of type $\{0, 0, a, a, a, b, c, c, d, d, d, 0\}$ with conditions $\{c \leq 1/10\}$ has maximum in $\{0.417778, \{a > 0, b > 1, c > 0.1, d > 0.266667\}\}$

Point number 4.2.2 of type $\{0, 0, a, a, a, b, 0, c, d, d, d, 0\}$ with conditions $\{\}$ has maximum in $\{0.384, \{a > 0.2, b > 0.4, c > 0.4, d > 0.2\}\}$

Point number 4.2.3 of type $\{0, 0, a, a, a, b, 1/10, c, d, d, d, 0\}$ with conditions $\{\}$ has maximum in $\{0.474727, \{a > 0.202922, b > 0.391233, c > 0.36452, d > 0.178493\}\}$

Point number 4.3.1 of type $\{1/10, 0, 0, 0, 0, b, s_2, c, c, c, c, 0\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.4374, \{s_2 > 0.1, b > 0.9, c > 0.225\}\}$

Point number 4.3.2 of type $\{1/10, 0, 3/10, 3/10, 3/10, 0, s_2, c, d, d, d, 0\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.346954, \{s_2 > 0.1, c > 0.484325, d > 0.138558\}\}$

Point number 4.3.3 of type $\{1/10, 0, a, a, a, b, s_2, 0, (1-s_2)/3, (1-s_2)/3, (1-s_2)/3, 0\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.346954, \{s_2 > 0.1, a > 0.138558, b > 0.484325\}\}$

Point number 4.3.4 of type $\{1/10, 0, 9/40, 9/40, 9/40, 9/40, s_2, 1-s_2, 0, 0, 0, 0\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.455625, \{s_2 > 0\}\}$

Point number 4.3.5 of type $\{1/10, 0, a, a, a, b, 1/10, c, d, d, d, 0\}$ with conditions $\{\}$ has maximum in $\{0.531878, \{a > 0.18, b > 0.36, c > 0.36, d > 0.18\}\}$

Point number 4.3.6 of type $\{1/10, 0, a, a, a, b, 0, c, d, d, d, 0\}$ with conditions $\{\}$ has maximum in $\{0.474727, \{a > 0.178493, b > 0.36452, c > 0.391233, d > 0.202922\}\}$

Point number 5.1 of type $\{a, 0, a, a, b, b, d, c, d, d, 0, 0\}$ with conditions

$\{a \leq 1/10, d \leq 1/10\}$ has maximum in $\{0.40845, \{a > 0.1, b > 0.35, c > 0.7, d > 0.1\}\}$

Point number 5.2.1 of type $\{\{0, 0, a, a, b, b\}, \{c, c, d, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.522114, \{a > 0.183013, b > 0.316987, c > 0.316987, d > 0.183013\}\}$

Point number 5.2.2 of type $\{\{0, 0, a, a, b, b\}, \{0, c, d, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.375, \{a > 0.25, b > 0.25, c > 1., d > 0.\}\}$

Point number 5.2.3 of type $\{\{0, 0, a, a, b, b\}, \{1/10, c, d, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.447387, \{a > 0.169923, b > 0.330077, c > 0.414246, d > 0.242877\}\}$

Point number 5.3.1 of type $\{\{1/10, 0, 0, 0, b, b\}, \{s_2, c, c, c, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.41715, \{s_2 > 0.1, b > 0.45, c > 0.3\}\}$

Point number 5.3.2 of type $\{\{1/10, 0, a, a, 0, 0\}, \{s_2, c, d, d, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.1296, \{s_2 > 0.1, a > 0.45, c > 0.6, d > 0.15\}\}$

Point number 5.3.3 of type $\{\{1/10, 0, a, a, b, b\}, \{s_2, 0, d, d, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.274316, \{s_2 > 0.1, a > 0.10793, b > 0.34207, d > 0.45\}\}$

Point number 5.3.4 of type $\{\{1/10, 0, a, a, a, a\}, \{s_2, c, 0, 0, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.455625, \{s_2 > 0., a > 0.225, c > 1.\}\}$

Point number 5.3.5 of type $\{\{1/10, 0, a, a, b, b\}, \{1/10, c, d, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.504311, \{a > 0.139658, b > 0.310342, c > 0.403687, d > 0.248156\}\}$

Point number 5.3.6 of type $\{\{1/10, 0, a, a, b, b\}, \{0, c, d, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.455625, \{a > 0.225, b > 0.225, c > 1., d > 0.\}\}$

Point number 6.1 of type $\{\{a, 0, 0, a, b, b\}, \{d, c, c, d, 0, 0\}\}$ with conditions $\{a \leq 1/10, d \leq 1/10\}$ has maximum in $\{0.4224, \{a > 0.1, b > 0.4, c > 0.4, d > 0.1\}\}$

Point number 6.2.1 of type $\{\{0, 0, 0, a, b, b\}, \{c, c, c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.47841, \{a > 0.257041, b > 0.37148, c > 0.283317, d > 0.15005\}\}$

Point number 6.2.2 of type $\{\{0, 0, 0, a, b, b\}, \{0, c, c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.316406, \{a > 0.25, b > 0.375, c > 0.375, d > 0.25\}\}$

Point number 6.2.3 of type $\{\{0, 0, 0, a, b, b\}, \{1/10, c, c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.415799, \{a > 0.24392, b > 0.37804, c > 0.343017, d > 0.213966\}\}$

Point number 6.3.1 of type $\{\{1/10, 0, 0, 0, b, b\}, \{s_2, c, c, c, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.41715, \{s_2 > 0.1, b > 0.45, c > 0.3\}\}$

Point number 6.3.2 of type $\{\{1/10, 0, 0, a, 0, 0\}, \{s_2, c, c, d, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.10935, \{s_2 > 0.1, a > 0.9, c > 0.45, d > 0.\}\}$

Point number 6.3.3 of type $\{\{1/10, 0, 0, a, b, b\}, \{s_2, 0, 0, d, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.1215, \{s_2 > 0., a > 0., b > 0.45, d > 1.\}\}$

Point number 6.3.4 of type $\{\{1/10, 0, 0, a, a, a\}, \{s_2, c, c, 0, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10\}$ has maximum in $\{0.41715, \{s_2 > 0.1, a > 0.3, c > 0.45\}\}$

Point number 6.3.5 of type $\{\{1/10, 0, 0, a, b, b\}, \{1/10, c, c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.485482, \{a > 0.209167, b > 0.345416, c > 0.345416, d > 0.209167\}\}$

Point number 6.3.6 of type $\{\{1/10, 0, 0, a, b, b\}, \{0, c, c, d, 0, 0\}\}$ with conditions $\{\}$ has maximum in $\{0.415799, \{a > 0.213966, b > 0.343017, c > 0.37804, d > 0.24392\}\}$

Point number 7 of type $\{\{s_1, 0, (1-s_1)/4, (1-s_1)/4, (1-s_1)/4, (1-s_1)/4\}, \{0, 1, 0, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10\}$ has maximum in $\{0.455625, \{s_1 > 0.1\}\}$

Point number 8 of type $\{\{s_1, 0, 0, (1-s_1)/3, (1-s_1)/3, (1-s_1)/3\}, \{0, 1, 0, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10\}$ has maximum in $\{0.324, \{s_1 > 0.1\}\}$

Point number 9 of type $\{\{s_1, a, a, b, b, b\}, \{s_2, (1-s_2)/2, (1-s_2)/2, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10, s_2 \leq 1/10\}$ has maximum in $\{0.421414, \{s_1 > 0.1, s_2 > 0.1, a > 0.0292038, b > 0.280531\}\}$

Point number 10 of type $\{\{0, 0, 1-3b, b, b, b\}, \{s_2, c, 1-c-s_2, 0, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10, 1-3b \geq 0, 1-c-s_2 \geq 0\}$ has maximum in $\{0.385969, \{s_2 > 0.1, b > 0.264433, c > 0.723293, a > 0.5\}\}$

Point number 11 of type $\{\{1/10, 0, 9/10-3b, b, b, b\}, \{s_2, c, 1-c-s_2, 0, 0, 0\}\}$ with conditions $\{s_2 \leq 1/10, 9/10-3b \geq 0, 1-c-s_2 \geq 0\}$ has maximum in $\{0.455625, \{s_2 > 0.1, b > 0.225, c > 1.\}\}$

Point number 12 of type $\{\{s_1, 0, 0, b, b, b\}, \{s_2, c, c, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10, s_2 \leq 1/10\}$ has maximum in $\{0.41715, \{s_1 > 0.1, s_2 > 0.1, b > 0.3, c > 0.45\}\}$

Point number 13 of type $\{\{s_1, 0, 0, b, b, b\}, \{s_2, c, c, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10, s_2 \leq 1/10\}$ has maximum in $\{0.41715, \{s_1 > 0.1, s_2 > 0.1, b > 0.3, c > 0.45\}\}$

Point number 14 of type $\{\{s_1, 0, 1-s_1, 0, 0, 0\}, \{s_2, c, 1-c-s_2, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10, s_2 \leq 1/10\}$ has maximum in $\{0., \{s_1 > 0., s_2 > 0., c > 0.\}\}$

Point number 15 of type $\{\{s_1, 0, 1-3b-s_1, b, b, b\}, \{0, c, 1-c, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10, 1-3b-s_1 \geq 0, 1-c \geq 0\}$ has maximum in $\{0.455625, \{s_1 > 0.1, b > 0.225, c > 1., a > 0.5\}\}$

Point number 16 of type $\{\{s_1, 0, 1-3b-s_1, b, b, b\}, \{1/10, c, 9/10-c, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10, 1-3b-s_1 \geq 0, 9/10-c \geq 0\}$ has maximum in $\{0.441899, \{s_1 > 0.1, b > 0.237247, c > 0.767449, a > 0.5\}\}$

Point number 17 of type $\{\{s_1, 0, 1-3b-s_1, b, b, b\}, \{1/10, c, 0, 0, 0, 0\}\}$ with conditions $\{s_1 \leq 1/10, 1-3b-s_1 \geq 0\}$ has maximum in $\{0.4374, \{s_1 > 0.1, b > 0.225, c > 0.9, a > 0.5\}\}$

Max attained in point 2 with value 0.5351057976970544`.

EXACT VALUE OF M_4 (Unbalanced-Unbalanced distributions on different

coordinates)

Point number 1 of type $\{\{a_1, a_1, a_1, a_1, a_1, a_1\}, \{b_1, b_1, b_1, b_1, b_1, b_1\}\}$ with value $\{0.555556, \{a_1 \rightarrow 0.166667, b_1 \rightarrow 0.166667\}\}$

Point number 2 of type $\{\{0, a_1, a_1, a_1, a_1, a_1\}, \{1-5 b_1, b_1, b_1, b_1, b_1, b_1\}\}$ with value $\{0.558469, \{a_1 \rightarrow 0.2, b_1 \rightarrow 0.130217\}\}$

Point number 3 of type $\{\{0, a_1, a_1, a_1, a_1, 1-4 a_1\}, \{1-4 b_1, b_1, b_1, b_1, b_1, 0\}\}$ with value $\{0.555556, \{a_1 \rightarrow 0.166667, b_1 \rightarrow 0.166667\}\}$

Point number 4 of type $\{\{0, 0, a_1, a_1, a_1, a_1\}, \{1/2 (1-4 b_1), 1/2 (1-4 b_1), b_1, b_1, b_1, b_1\}\}$ with value $\{0.538531, \{a_1 \rightarrow 0.25, b_1 \rightarrow 0.0986122\}\}$

Point number 5 of type $\{\{0, 0, a_1, a_1, a_1, 1-3 a_1\}, \{1/2 (1-3 b_1), 1/2 (1-3 b_1), b_1, b_1, b_1, 0\}\}$ with value $\{0.536479, \{a_1 \rightarrow 0.217029, b_1 \rightarrow 0.133324\}\}$

Point number 6 of type $\{\{0, 0, a_1, a_1, 1/2 (1-2 a_1), 1/2 (1-2 a_1)\}, \{1/2 (1-2 b_1), 1/2 (1-2 b_1), b_1, b_1, 0, 0\}\}$ with value $\{0.522114, \{a_1 \rightarrow 0.183013, b_1 \rightarrow 0.183013\}\}$

Point number 7 of type $\{\{0, 0, 0, a_1, a_1, a_1\}, \{1/3 (1-3 b_1), 1/3 (1-3 b_1), 1/3 (1-3 b_1), b_1, b_1, b_1\}\}$ with value $\{0.48385, \{a_1 \rightarrow 0.333333, b_1 \rightarrow 0.0611678\}\}$

Point number 8 of type $\{\{0, 0, 0, a_1, a_1, 1-2 a_1\}, \{1/3 (1-2 b_1), 1/3 (1-2 b_1), 1/3 (1-2 b_1), b_1, b_1, 0\}\}$ with value $\{0.485279, \{a_1 \rightarrow 0.292851, b_1 \rightarrow 0.0972416\}\}$

Point number 9 of type $\{\{0, 0, 0, a_1, (1-a_1)/2, (1-a_1)/2\}, \{(1-b_1)/3, (1-b_1)/3, (1-b_1)/3, b_1, 0, 0\}\}$ with value $\{0.47841, \{a_1 \rightarrow 0.257041, b_1 \rightarrow 0.15005\}\}$

Point number 10 of type $\{\{0, 0, 0, 1/3, 1/3, 1/3\}, \{1/3, 1/3, 1/3, 0, 0, 0\}\}$ with value $\{0.444444, \{a_1 \rightarrow 0., b_1 \rightarrow 46.\}\}$

Max attained in point 2 with value 0.5584691723152507`.

UPPER BOUND ON THE RATE

The bound on the rate for $b=6$, $k=5$ and $\epsilon=1/10$ is 0.3451128106351136` and $\eta_0=0.6216033003895701`$