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Title: Mathematica Output for the upper bound when $B=14$, $K=12$ and $\epsilon=1/13$

EXACT VALUE OF M_1 (Balanced-Balanced distributions)

Point number 1 of type $\{\{a,a,a,a,a,a,a,a,a,a,a,a,a\},\{d,d,d,d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00717594,\{a>0.0714286,d->0.0714286\}\}$

Point number 2 of type $\{\{0,a,a,a,a,a,a,a,a,a,a,a\},\{1-13 d,d,d,d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00708282,\{a>0.0769231,d->0.00591716\}\}$

Point number 3 of type $\{\{0,a,a,a,a,a,a,a,a,a,a,a,1-12 a\},\{1-12 d,d,d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00708283,\{a>0.00641026,d->0.0769097\}\}$

Point number 4 of type $\{\{0,0,a,a,a,a,a,a,a,a,a,a\},\{1/2 (1-12 d),1/2 (1-12 d),d,d,d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00478941,\{a>0.0833333,d->0.068834\}\}$

Point number 5 of type $\{\{0,0,a,a,a,a,a,a,a,a,a,a,1-11 a\},\{1/2 (1-11 d),1/2 (1-11 d),d,d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00708284,\{a>0.00699301,d->0.0768938\}\}$

Point number 6 of type $\{\{0,0,a,a,a,a,a,a,a,a,a,a,1/2 (1-10 a),1/2 (1-10 a)\},\{1/2 (1-10 d),1/2 (1-10 d),d,d,d,d,d,d,d,d,d,d,0,0\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00362013,\{a>0.00769231,d->0.083308\}\}$

Point number 7 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a\},\{1/3 (1-11 d),1/3 (1-11 d),1/3 (1-11 d),d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.0041029,\{a>0.0909091,d->0.0685887\}\}$

Point number 8 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a,1-10 a\},\{1/3 (1-10 d),1/3 (1-10 d),1/3 (1-10 d),d,d,d,d,d,d,d,d,d,d,0\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00708285,\{a>0.00769231,d->0.0768748\}\}$

Point number 9 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a,1/2 (1-9 a),1/2 (1-9 a)\},\{1/3 (1-9 d),1/3 (1-9 d),1/3 (1-9 d),d,d,d,d,d,d,d,d,d,d,0,0\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00362013,\{a>0.00854701,d->0.0832911\}\}$

Point number 10 of type $\{\{0,0,0,a,a,a,a,a,a,a,a,a,a,1/3 (1-8 a),1/3 (1-8 a),1/3 (1-8 a)\},\{1/3 (1-8 d),1/3 (1-8 d),1/3 (1-8 d),d,d,d,d,d,d,d,d,d,d,0,0,0\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00143135,\{a>0.00961538,d->0.0908806\}\}$

Point number 11 of type $\{\{0,0,0,0,a,a,a,a,a,a,a,a,a,a\},\{1/4 (1-10 d),1/4 (1-10 d),1/4 (1-10 d),1/4 (1-10 d),d,d,d,d,d,d,d,d,d,d\}\}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00375896,\{a>0.1,d->0.0682189\}\}$

Point number 12 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,1-9 a\}, \{1/4 (1-9 d), 1/4 (1-9 d), 1/4 (1-9 d), 1/4 (1-9 d), d,d,d,d,d,d,d,d,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00708287, \{a \rightarrow 0.00854701, d \rightarrow 0.0768515\} \}$

Point number 13 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,1/2 (1-8 a), 1/2 (1-8 a)\}, \{1/4 (1-8 d), 1/4 (1-8 d), 1/4 (1-8 d), 1/4 (1-8 d), d,d,d,d,d,d,d,d,0,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00362014, \{a \rightarrow 0.00961538, d \rightarrow 0.08327\} \}$

Point number 14 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,1/3 (1-7 a), 1/3 (1-7 a), 1/3 (1-7 a)\}, \{1/4 (1-7 d), 1/4 (1-7 d), 1/4 (1-7 d), 1/4 (1-7 d), d,d,d,d,d,d,d,d,0,0,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00143135, \{a \rightarrow 0.010989, d \rightarrow 0.0908657\} \}$

Point number 15 of type $\{ \{0,0,0,0,a,a,a,a,a,a,a,1/4 (1-6 a), 1/4 (1-6 a), 1/4 (1-6 a), 1/4 (1-6 a)\}, \{1/4 (1-6 d), 1/4 (1-6 d), 1/4 (1-6 d), 1/4 (1-6 d), d,d,d,d,d,d,d,d,0,0,0,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.000334984, \{a \rightarrow 0.0128205, d \rightarrow 0.0999995\} \}$

Point number 16 of type $\{ \{12/13,a,a,a,a,a,a,a,a,a,a,a,a\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00708282, \{a \rightarrow 0.00591716, d \rightarrow 0.0769231\} \}$

Point number 17 of type $\{ \{12/13,a,a,a,a,a,a,a,a,a,a,a,a,1/13-12 a\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00386807, \{a \rightarrow 0., d \rightarrow 0.0833333\} \}$

Point number 18 of type $\{ \{12/13,a,a,a,a,a,a,a,a,a,a,a,a,1/2 (1/13-11 a), 1/2 (1/13-11 a)\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d,0,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00153897, \{a \rightarrow 0., d \rightarrow 0.0909091\} \}$

Point number 19 of type $\{ \{12/13,a,a,a,a,a,a,a,a,a,a,a,a,1/3 (1/13-10 a), 1/3 (1/13-10 a), 1/3 (1/13-10 a)\}, \{0,d,d,d,d,d,d,d,d,d,d,d,d,0,0,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00036288, \{a \rightarrow 0., d \rightarrow 0.1\} \}$

Point number 20 of type $\{ \{12/13,0,a,a,a,a,a,a,a,a,a,a,a,a\}, \{0,1-12 d,d,d,d,d,d,d,d,d,d,d,d,d\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00708283, \{a \rightarrow 0.00641026, d \rightarrow 0.0769097\} \}$

Point number 21 of type $\{ \{12/13,0,a,a,a,a,a,a,a,a,a,a,a,a,1/13-11 a\}, \{0,1-11 d,d,d,d,d,d,d,d,d,d,d,d,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00386807, \{a \rightarrow 0., d \rightarrow 0.0833333\} \}$

Point number 22 of type $\{ \{12/13,0,a,a,a,a,a,a,a,a,a,a,a,a,1/2 (1/13-10 a), 1/2 (1/13-10 a)\}, \{0,1-10 d,d,d,d,d,d,d,d,d,d,d,d,0,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00153897, \{a \rightarrow 0., d \rightarrow 0.0909091\} \}$

Point number 23 of type $\{ \{12/13,0,a,a,a,a,a,a,a,a,a,a,a,a,1/3 (1/13-9 a), 1/3 (1/13-9 a), 1/3 (1/13-9 a)\}, \{0,1-9 d,d,d,d,d,d,d,d,d,d,d,d,0,0,0\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to $\{0.00036288, \{a \rightarrow 0., d \rightarrow 0.1\} \}$

Point number 24 of type $\{ \{12/13,0,0,a,a,a,a,a,a,a,a,a,a,a\}, \{0,1/2 (1-11 d), 1/2 (1-11 d), d,d,d,d,d,d,d,d,d,d,d,d\} \}$ with $\{a,d\} \leq 1/13$ has maximum value equal to

{0.00708284,{a->0.00699301,d->0.0768938}}

Point number 25 of type {{12/13,0,0,a,a,a,a,a,a,a,1/13-10 a},{0,1/2 (1-10 d),1/2 (1-10 d),d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/13 has maximum value equal to {0.00386807,{a->0.,d->0.0833333}}

Point number 26 of type {{12/13,0,0,a,a,a,a,a,a,a,1/2 (1/13-9 a),1/2 (1/13-9 a)}, {0,1/2 (1-9 d),1/2 (1-9 d),d,d,d,d,d,d,d,d,0,0}} with {a,d} <= 1/13 has maximum value equal to {0.00153897,{a->0.,d->0.0909091}}

Point number 27 of type {{12/13,0,0,a,a,a,a,a,a,a,1/3 (1/13-8 a),1/3 (1/13-8 a),1/3 (1/13-8 a)}, {0,1/2 (1-8 d),1/2 (1-8 d),d,d,d,d,d,d,d,d,0,0,0}} with {a,d} <= 1/13 has maximum value equal to {0.00036288,{a->0.,d->0.1}}

Point number 28 of type {{12/13,0,0,0,a,a,a,a,a,a,a,a},{0,1/3 (1-10 d),1/3 (1-10 d),1/3 (1-10 d),d,d,d,d,d,d,d,d,d}} with {a,d} <= 1/13 has maximum value equal to {0.00708285,{a->0.00769231,d->0.0768748}}

Point number 29 of type {{12/13,0,0,0,a,a,a,a,a,a,a,1/13-9 a},{0,1/3 (1-9 d),1/3 (1-9 d),1/3 (1-9 d),d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/13 has maximum value equal to {0.00386807,{a->0.,d->0.0833333}}

Point number 30 of type {{12/13,0,0,0,a,a,a,a,a,a,a,1/2 (1/13-8 a),1/2 (1/13-8 a)}, {0,1/3 (1-8 d),1/3 (1-8 d),1/3 (1-8 d),d,d,d,d,d,d,d,d,0,0}} with {a,d} <= 1/13 has maximum value equal to {0.00153897,{a->0.,d->0.0909091}}

Point number 31 of type {{12/13,0,0,0,a,a,a,a,a,a,a,1/3 (1/13-7 a),1/3 (1/13-7 a),1/3 (1/13-7 a)}, {0,1/3 (1-7 d),1/3 (1-7 d),1/3 (1-7 d),d,d,d,d,d,d,d,d,0,0,0}} with {a,d} <= 1/13 has maximum value equal to {0.00036288,{a->0.,d->0.1}}

Point number 32 of type {{12/13,0,0,0,0,a,a,a,a,a,a,a,a},{0,1/4 (1-9 d),1/4 (1-9 d),1/4 (1-9 d),1/4 (1-9 d),d,d,d,d,d,d,d,d}} with {a,d} <= 1/13 has maximum value equal to {0.00708287,{a->0.00854701,d->0.0768515}}

Point number 33 of type {{12/13,0,0,0,0,a,a,a,a,a,a,a,1/13-8 a},{0,1/4 (1-8 d),1/4 (1-8 d),1/4 (1-8 d),1/4 (1-8 d),d,d,d,d,d,d,d,d,0}} with {a,d} <= 1/13 has maximum value equal to {0.00386807,{a->0.,d->0.0833333}}

Point number 34 of type {{12/13,0,0,0,0,a,a,a,a,a,a,a,1/2 (1/13-7 a),1/2 (1/13-7 a)}, {0,1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),d,d,d,d,d,d,d,d,0,0}} with {a,d} <= 1/13 has maximum value equal to {0.00153897,{a->0.,d->0.0909091}}

Point number 35 of type {{12/13,0,0,0,0,a,a,a,a,a,a,a,1/3 (1/13-6 a),1/3 (1/13-6 a),1/3 (1/13-6 a)}, {0,1/4 (1-6 d),1/4 (1-6 d),1/4 (1-6 d),1/4 (1-6 d),d,d,d,d,d,d,d,d,0,0,0}} with {a,d} <= 1/13 has maximum value equal to {0.00036288,{a->0.,d->0.1}}

Point number 36 of type {{12/13,a,a,a,a,a,a,a,a,a,a,a,0}, {0,d,d,d,d,d,d,d,d,d,d,d,12/13}} with {a,d} <= 1/13 has maximum value equal to {2.54345*10^-11,{a->0.00641026,d->0.00641026}}

Point number 37 of type $\{\{12/13, 0, a, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/13-11 d, d, d, d, d, d, d, d, d, d, d, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{1.96707 \cdot 10^{-11}, \{a > 0.00699301, d > 0.00640579\}\}$

Point number 38 of type $\{\{12/13, 0, a, a, a, a, a, a, a, a, 1/13-10 a, 0\}, \{0, 1/13-10 d, d, d, d, d, d, d, d, d, d, 0, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{1.3914 \cdot 10^{-11}, \{a > 0.00698591, d > 0.00698591\}\}$

Point number 39 of type $\{\{12/13, 0, 0, a, a, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/13-10 d), 1/2 (1/13-10 d), d, d, d, d, d, d, d, d, d, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{1.56964 \cdot 10^{-11}, \{a > 0.00769231, d > 0.00640419\}\}$

Point number 40 of type $\{\{12/13, 0, 0, a, a, a, a, a, a, a, a, 1/13-9 a, 0\}, \{0, 1/2 (1/13-9 d), 1/2 (1/13-9 d), d, d, d, d, d, d, d, d, 0, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{9.94381 \cdot 10^{-12}, \{a > 0.00767642, d > 0.00698487\}\}$

Point number 41 of type $\{\{12/13, 0, 0, a, a, a, a, a, a, a, 1/2 (1/13-8 a), 1/2 (1/13-8 a), 0\}, \{0, 1/2 (1/13-8 d), 1/2 (1/13-8 d), d, d, d, d, d, d, d, d, 0, 0, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{5.97288 \cdot 10^{-12}, \{a > 0.00767659, d > 0.00767659\}\}$

Point number 42 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, a, a, 0\}, \{0, 1/3 (1/13-9 d), 1/3 (1/13-9 d), 1/3 (1/13-9 d), d, d, d, d, d, d, d, d, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{1.34857 \cdot 10^{-11}, \{a > 0.00854701, d > 0.00640361\}\}$

Point number 43 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, a, a, 1/13-8 a, 0\}, \{0, 1/3 (1/13-8 d), 1/3 (1/13-8 d), 1/3 (1/13-8 d), d, d, d, d, d, d, d, 0, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{7.73747 \cdot 10^{-12}, \{a > 0.00847973, d > 0.00698636\}\}$

Point number 44 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, a, 1/2 (1/13-7 a), 1/2 (1/13-7 a), 0\}, \{0, 1/3 (1/13-7 d), 1/3 (1/13-7 d), 1/3 (1/13-7 d), d, d, d, d, d, d, d, 0, 0, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{3.76316 \cdot 10^{-12}, \{a > 0.00848604, d > 0.00768368\}\}$

Point number 45 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, 1/3 (1/13-6 a), 1/3 (1/13-6 a), 1/3 (1/13-6 a), 0\}, \{0, 1/3 (1/13-6 d), 1/3 (1/13-6 d), 1/3 (1/13-6 d), d, d, d, d, d, d, 0, 0, 0, 12/13\}\}$ with $\{a, d\} \leq 1/13$ has maximum value equal to $\{1.54714 \cdot 10^{-12}, \{a > 0.00852127, d > 0.00852127\}\}$

Point number 46 of type $\{\{12/13, 1/13, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, a, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12, (1-a)/12\}\}$ with $\{a\} \leq 1/13$ has maximum value equal to $\{0.00708426, \{a > 0.0749465\}\}$

Point number 47 of type $\{\{12/13, 1/13, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12, 1/12\}\}$ with $\{ \} \leq 1/13$ has maximum value equal to $0.0038680716306584363`$

Point number 48 of type $\{\{12/13, 1/13, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11\}\}$ with $\{ \} \leq 1/13$ has maximum value equal to $0.0015389654375500732`$

Point number 49 of type $\{\{12/13, 1/13, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10, 1/10\}\}$ with $\{\} \leq 1/13$ has maximum value equal to 0.00036288`

Max attained in point 1 with value 0.007175938448327556`.

EXACT VALUE OF M_2 (Unbalanced-Balanced distributions)

Point number 1 of type $\{\{a, a, a, a, a, a, a, a, a, a, a, a, a\}, \{d, d, d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00717594, \{a \rightarrow 0.0714286, d \rightarrow 0.0714286\}\}$

Point number 2 of type $\{\{0, a, a, a, a, a, a, a, a, a, a, a, a\}, \{1-13 d, d, d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00752828, \{a \rightarrow 0.0769231, d \rightarrow 0.\}\}$

Point number 3 of type $\{\{0, a, a, a, a, a, a, a, a, a, a, a, 1-12 a\}, \{1-12 d, d, d, d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00752828, \{a \rightarrow 0., d \rightarrow 0.0769231\}\}$

Point number 4 of type $\{\{0, 0, a, a, a, a, a, a, a, a, a, a, a\}, \{1/2 (1-12 d), 1/2 (1-12 d), d, d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00478941, \{a \rightarrow 0.0833333, d \rightarrow 0.068834\}\}$

Point number 5 of type $\{\{0, 0, a, a, a, a, a, a, a, a, a, a, 1-11 a\}, \{1/2 (1-11 d), 1/2 (1-11 d), d, d, d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00752828, \{a \rightarrow 0., d \rightarrow 0.0769231\}\}$

Point number 6 of type $\{\{0, 0, a, a, a, a, a, a, a, a, a, 1/2 (1-10 a), 1/2 (1-10 a)\}, \{1/2 (1-10 d), 1/2 (1-10 d), d, d, d, d, d, d, d, d, d, 0, 0\}\}$ with value $\{0.00386807, \{a \rightarrow 0., d \rightarrow 0.0833333\}\}$

Point number 7 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, a, a, a\}, \{1/3 (1-11 d), 1/3 (1-11 d), 1/3 (1-11 d), d, d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.0041029, \{a \rightarrow 0.0909091, d \rightarrow 0.0685887\}\}$

Point number 8 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, a, a, 1-10 a\}, \{1/3 (1-10 d), 1/3 (1-10 d), 1/3 (1-10 d), d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00752828, \{a \rightarrow 0., d \rightarrow 0.0769231\}\}$

Point number 9 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, a, 1/2 (1-9 a), 1/2 (1-9 a)\}, \{1/3 (1-9 d), 1/3 (1-9 d), 1/3 (1-9 d), d, d, d, d, d, d, d, d, d, 0, 0\}\}$ with value $\{0.00386807, \{a \rightarrow 0., d \rightarrow 0.0833333\}\}$

Point number 10 of type $\{\{0, 0, 0, a, a, a, a, a, a, a, a, 1/3 (1-8 a), 1/3 (1-8 a), 1/3 (1-8 a)\}, \{1/3 (1-8 d), 1/3 (1-8 d), 1/3 (1-8 d), d, d, d, d, d, d, d, d, d, 0, 0\}\}$ with value $\{0.00153897, \{a \rightarrow 0., d \rightarrow 0.0909091\}\}$

Point number 11 of type $\{\{0, 0, 0, 0, a, a, a, a, a, a, a, a, a\}, \{1/4 (1-10 d), 1/4 (1-10 d), 1/4 (1-10 d), 1/4 (1-10 d), d, d, d, d, d, d, d, d, d, d\}\}$ with value $\{0.00375896, \{a \rightarrow 0.1, d \rightarrow 0.0682189\}\}$

Point number 12 of type $\{\{0, 0, 0, 0, a, a, a, a, a, a, a, a, 1-9 a\}, \{1/4 (1-9 d), 1/4 (1-9 d), 1/4 (1-9 d), 1/4 (1-9 d), d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00752828, \{a \rightarrow 0., d \rightarrow 0.0769231\}\}$

Point number 13 of type $\{\{0, 0, 0, 0, a, a, a, a, a, a, a, 1/2 (1-8 a), 1/2 (1-8 a)\}, \{1/4 (1-8 d), 1/4 (1-8 d), 1/4 (1-8 d), 1/4 (1-8 d), d, d, d, d, d, d, d, d, d, 0\}\}$ with value $\{0.00386807, \{a \rightarrow 0., d \rightarrow 0.068834\}\}$

>0.,d->0.0833333}}

Point number 14 of type {{0,0,0,0,a,a,a,a,a,a,1/3 (1-7 a),1/3 (1-7 a),1/3 (1-7 a)},1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),1/4 (1-7 d),d,d,d,d,d,d,0,0,0}} with value {0.00153897,{a->0.,d->0.0909091}}

Point number 15 of type {{0,0,0,0,a,a,a,a,a,a,1/4 (1-6 a),1/4 (1-6 a),1/4 (1-6 a),1/4 (1-6 a)},1/4 (1-6 d),1/4 (1-6 d),1/4 (1-6 d),1/4 (1-6 d),d,d,d,d,d,d,0,0,0}} with value {0.00036288,{a->0.,d->0.1}}

Max attained in point 2 with value 0.00752827617711624`.

EXACT VALUE OF M_3 (Unbalanced-Unbalanced distributions on the same coordinate)

Point number 1 of type {{12/13,a,a,a,a,a,a,a,a,a,a,a},12/13,d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {1.11092*10⁻¹²,{a->0.00591716,d->0.00591716}}

Point number 2 of type {{12/13,0,a,a,a,a,a,a,a,a,a,a,a},12/13,1/13-12 d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {1.06406*10⁻¹²,{a->0.00641026,d->0.}}

Point number 3 of type {{12/13,0,a,a,a,a,a,a,a,a,a,a,a,1/13-11 a},12/13,1/13-11 d,d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {1.06406*10⁻¹²,{a->0.,d->0.00641026}}

Point number 4 of type {{12/13,0,0,a,a,a,a,a,a,a,a,a,a,a},12/13,1/2 (1/13-11 d),1/2 (1/13-11 d),d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {7.55354*10⁻¹³,{a->0.00699301,d->0.00567773}}

Point number 5 of type {{12/13,0,0,a,a,a,a,a,a,a,a,a,a,a,1/13-10 a},12/13,1/2 (1/13-10 d),1/2 (1/13-10 d),d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {1.06406*10⁻¹²,{a->0.,d->0.00641026}}

Point number 6 of type {{12/13,0,0,a,a,a,a,a,a,a,a,a,a,1/2 (1/13-9 a),1/2 (1/13-9 a)},12/13,1/2 (1/13-9 d),1/2 (1/13-9 d),d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {5.77919*10⁻¹³,{a->0.,d->0.00699301}}

Point number 7 of type {{12/13,0,0,0,a,a,a,a,a,a,a,a,a,a,a},12/13,1/3 (1/13-10 d),1/3 (1/13-10 d),1/3 (1/13-10 d),d,d,d,d,d,d,d,d,d,d,d}} has maximum value equal to {6.44891*10⁻¹³,{a->0.00769231,d->0.00564839}}

Point number 8 of type {{12/13,0,0,0,a,a,a,a,a,a,a,a,a,a,a,1/13-9 a},12/13,1/3 (1/13-9 d),1/3 (1/13-9 d),1/3 (1/13-9 d),d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {1.06406*10⁻¹²,{a->0.,d->0.00641026}}

Point number 9 of type {{12/13,0,0,0,a,a,a,a,a,a,a,a,a,a,a,1/2 (1/13-8 a),1/2 (1/13-8 a)},12/13,1/3 (1/13-8 d),1/3 (1/13-8 d),1/3 (1/13-8 d),d,d,d,d,d,d,d,d,d,d,0}} has maximum value equal to {5.77919*10⁻¹³,{a->0.,d->0.00699301}}

Point number 10 of type $\{\{12/13,0,0,0,a,a,a,a,a,1/3 (1/13-7 a),1/3 (1/13-7 a),1/3 (1/13-7 a)\},\{12/13,1/3 (1/13-7 d),1/3 (1/13-7 d),1/3 (1/13-7 d),d,d,d,d,d,d,0,0,0\}\}$ has maximum value equal to $\{2.45611*10^{-13},\{a>0.,d>0.00769231\}\}$

Point number 11 of type $\{\{12/13,0,0,0,0,a,a,a,a,a,a,a\},\{12/13,1/4 (1/13-9 d),1/4 (1/13-9 d),1/4 (1/13-9 d),1/4 (1/13-9 d),d,d,d,d,d,d,d,d\}\}$ has maximum value equal to $\{5.85489*10^{-13},\{a>0.00854701,d>0.00561434\}\}$

Point number 12 of type $\{\{12/13,0,0,0,0,a,a,a,a,a,a,1/13-8 a\},\{12/13,1/4 (1/13-8 d),1/4 (1/13-8 d),1/4 (1/13-8 d),1/4 (1/13-8 d),d,d,d,d,d,d,d,0\}\}$ has maximum value equal to $\{1.06406*10^{-12},\{a>0.,d>0.00641026\}\}$

Point number 13 of type $\{\{12/13,0,0,0,0,a,a,a,a,a,a,1/2 (1/13-7 a),1/2 (1/13-7 a)\},\{12/13,1/4 (1/13-7 d),1/4 (1/13-7 d),1/4 (1/13-7 d),1/4 (1/13-7 d),d,d,d,d,d,d,d,0,0\}\}$ has maximum value equal to $\{5.77919*10^{-13},\{a>0.,d>0.00699301\}\}$

Point number 14 of type $\{\{12/13,0,0,0,0,a,a,a,a,a,1/3 (1/13-6 a),1/3 (1/13-6 a),1/3 (1/13-6 a)\},\{12/13,1/4 (1/13-6 d),1/4 (1/13-6 d),1/4 (1/13-6 d),1/4 (1/13-6 d),d,d,d,d,d,d,d,0,0,0\}\}$ has maximum value equal to $\{2.45611*10^{-13},\{a>0.,d>0.00769231\}\}$

Point number 15 of type $\{\{12/13,0,0,0,0,a,a,a,a,a,1/4 (1/13-5 a),1/4 (1/13-5 a),1/4 (1/13-5 a),1/4 (1/13-5 a)\},\{12/13,1/4 (1/13-5 d),1/4 (1/13-5 d),1/4 (1/13-5 d),1/4 (1/13-5 d),d,d,d,d,d,d,d,0,0,0,0\}\}$ has maximum value equal to $\{6.27169*10^{-14},\{a>0.,d>0.00854701\}\}$

Point number 16 of type $\{\{12/13,1/13,0,0,0,0,0,0,0,0,0,0,0\},\{12/13,a,1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a),1/12 (1/13-a)\}\}$ has maximum value equal to $\{1.06406*10^{-12},\{a>0.\}\}$

Point number 17 of type $\{\{12/13,1/13,0,0,0,0,0,0,0,0,0,0,0\},\{12/13,0,1/156,1/156,1/156,1/156,1/156,1/156,1/156,1/156,1/156,1/156,1/156\}\}$ has maximum value equal to $1.064056163867685*10^{-12}$

Point number 18 of type $\{\{12/13,1/13,0,0,0,0,0,0,0,0,0,0,0\},\{12/13,0,0,1/143,1/143,1/143,1/143,1/143,1/143,1/143,1/143,1/143,1/143\}\}$ has maximum value equal to $5.779191097612306*10^{-13}$

Point number 19 of type $\{\{12/13,1/13,0,0,0,0,0,0,0,0,0,0,0\},\{12/13,0,0,0,1/130,1/130,1/130,1/130,1/130,1/130,1/130,1/130,1/130,1/130\}\}$ has maximum value equal to $2.4561051648310916*10^{-13}$

Point number 20 of type $\{\{12/13,1/13,0,0,0,0,0,0,0,0,0,0,0\},\{12/13,0,0,0,0,1/117,1/117,1/117,1/117,1/117,1/117,1/117,1/117\}\}$ has maximum value equal to $6.271693392176687*10^{-14}$

Max attained in point 1 with value $1.1109159126537457*10^{-12}$.

EXACT VALUE OF M_4 (Unbalanced-Unbalanced distributions on different coordinates)

Point number 1 of type $\{\{12/13, a, a, a, a, a, a, a, a, a, a, a, 0\}, \{0, d, d, d, d, d, d, d, d, d, d, d, 12/13\}\}$ has maximum value equal to $\{2.54345 \cdot 10^{-11}, \{a \rightarrow 0.00641026, d \rightarrow 0.00641026\}\}$

Point number 2 of type $\{\{12/13, 0, a, a, a, a, a, a, a, a, a, 0\}, \{0, 1/13-11 d, d, d, d, d, d, d, d, d, d, d, 12/13\}\}$ has maximum value equal to $\{1.96707 \cdot 10^{-11}, \{a \rightarrow 0.00699301, d \rightarrow 0.00640579\}\}$

Point number 3 of type $\{\{12/13, 0, a, a, a, a, a, a, a, a, 1/13-10 a, 0\}, \{0, 1/13-10 d, d, d, d, d, d, d, d, d, d, 0, 12/13\}\}$ has maximum value equal to $\{1.3914 \cdot 10^{-11}, \{a \rightarrow 0.00698591, d \rightarrow 0.00698591\}\}$

Point number 4 of type $\{\{12/13, 0, 0, a, a, a, a, a, a, a, a, 0\}, \{0, 1/2 (1/13-10 d), 1/2 (1/13-10 d), d, d, d, d, d, d, d, d, d, 12/13\}\}$ has maximum value equal to $\{1.56964 \cdot 10^{-11}, \{a \rightarrow 0.00769231, d \rightarrow 0.00640419\}\}$

Point number 5 of type $\{\{12/13, 0, 0, a, a, a, a, a, a, a, a, 1/13-9 a, 0\}, \{0, 1/2 (1/13-9 d), 1/2 (1/13-9 d), d, d, d, d, d, d, d, d, 0, 12/13\}\}$ has maximum value equal to $\{9.94381 \cdot 10^{-12}, \{a \rightarrow 0.00767642, d \rightarrow 0.00698487\}\}$

Point number 6 of type $\{\{12/13, 0, 0, a, a, a, a, a, a, a, a, 1/2 (1/13-8 a), 1/2 (1/13-8 a), 0\}, \{0, 1/2 (1/13-8 d), 1/2 (1/13-8 d), d, d, d, d, d, d, d, d, 0, 12/13\}\}$ has maximum value equal to $\{5.97288 \cdot 10^{-12}, \{a \rightarrow 0.00767659, d \rightarrow 0.00767659\}\}$

Point number 7 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, a, a, a, 0\}, \{0, 1/3 (1/13-9 d), 1/3 (1/13-9 d), 1/3 (1/13-9 d), d, d, d, d, d, d, d, d, 12/13\}\}$ has maximum value equal to $\{1.34857 \cdot 10^{-11}, \{a \rightarrow 0.00854701, d \rightarrow 0.00640361\}\}$

Point number 8 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, a, a, a, 1/13-8 a, 0\}, \{0, 1/3 (1/13-8 d), 1/3 (1/13-8 d), 1/3 (1/13-8 d), d, d, d, d, d, d, d, d, 0, 12/13\}\}$ has maximum value equal to $\{7.73747 \cdot 10^{-12}, \{a \rightarrow 0.00847973, d \rightarrow 0.00698636\}\}$

Point number 9 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, a, a, a, 1/2 (1/13-7 a), 1/2 (1/13-7 a), 0\}, \{0, 1/3 (1/13-7 d), 1/3 (1/13-7 d), 1/3 (1/13-7 d), d, d, d, d, d, d, d, d, 0, 12/13\}\}$ has maximum value equal to $\{3.76316 \cdot 10^{-12}, \{a \rightarrow 0.00848604, d \rightarrow 0.00768368\}\}$

Point number 10 of type $\{\{12/13, 0, 0, 0, a, a, a, a, a, a, 1/3 (1/13-6 a), 1/3 (1/13-6 a), 1/3 (1/13-6 a), 0\}, \{0, 1/3 (1/13-6 d), 1/3 (1/13-6 d), 1/3 (1/13-6 d), d, d, d, d, d, d, d, d, 0, 12/13\}\}$ has maximum value equal to $\{1.54714 \cdot 10^{-12}, \{a \rightarrow 0.00852127, d \rightarrow 0.00852127\}\}$

Point number 11 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 1/156, 12/13\}\}$ has maximum value equal to $1.3496030816763965 \cdot 10^{-11}$

Point number 12 of type $\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1/143, 1/143, 1/143, 1/143, 1/143, 1/143, 1/143, 1/143, 1/143, 1/143, 1/143, 12/13\}\}$ has maximum value equal to $7.378987979462489 \cdot 10^{-12}$

Point number 13 of type $\{1,0,0,0,0,0,0,0,0,0,0,0,0\}$,
 $\{0,0,0,1/130,1/130,1/130,1/130,1/130,1/130,1/130,1/130,1/130,12/13\}$ has
maximum value equal to $3.1613495415093022 \times 10^{-12}$

Point number 14 of type $\{1,0,0,0,0,0,0,0,0,0,0,0,0\}$,
 $\{0,0,0,0,1/117,1/117,1/117,1/117,1/117,1/117,1/117,1/117,1/117,12/13\}$ has
maximum value equal to $8.153201409829693 \times 10^{-13}$

Max attained in point 1 with value $2.5434467587027902 \times 10^{-11}$.

UPPER BOUND ON THE RATE

The bound on the rate for $b=14$, $k=12$ and $\epsilon=1/13$ is 0.0071114574944462685
and $\eta_0=0.9552905732926196$