Lab 1

Local Averaging

$$A : \mathcal{L}^2(\mathbb{R}) \longrightarrow \ell^2(\mathbb{Z})$$

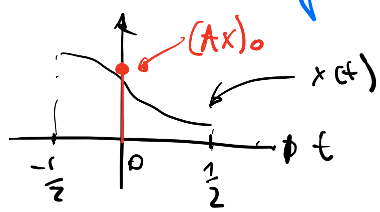
space of square summable functions

space of square summable sequences

such that given an $x(t) \in \mathcal{L}^2(\mathbb{R})$

$$(Ax)_n = \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} x(t)\, dt \qquad n \in \mathbb{Z}$$

mean value in interval of length 1



A is linear since the integral it is.

$Ax \in \ell^2(\mathbb{Z})$ since

$$\|Ax\|^2_{\ell^2(\mathbb{Z})} = \sum_{n\in\mathbb{Z}} |(Ax)_n|^2$$

$$= \sum_{n\in\mathbb{Z}} \left| \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} x(t)\, dt \right|^2$$

$$\leq \sum_{n\in\mathbb{Z}} \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} |x(t)|^2\, dt$$

non-overlapping intervals that span $(-\infty, +\infty)$

$$= \int_{-\infty}^{+\infty} |x(t)|^2\, dt = \|x\|^2_{\mathcal{L}^2(\mathbb{R})}$$

ADJOINT

$\forall\ x(t) \in L^2(\mathbb{R})\ ,\ y \in \ell^2(\mathbb{Z})$

$$\langle Ax, y \rangle_{\ell^2(\mathbb{Z})} = \langle x, A^* y \rangle_{L^2(\mathbb{R})}$$

where A^* is the adjoint operator.

$$\langle Ax, y \rangle_{\ell^2(\mathbb{Z})} = \sum_{n\in\mathbb{Z}} (Ax)_n\, y_n^* = \sum_{n\in\mathbb{Z}} \left(\int_{n-\frac{1}{2}}^{n+\frac{1}{2}} x(t)\, dt \right) y_n^*$$

$$= \sum_{n\in\mathbb{Z}} \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} x(t)\, \underbrace{y_n^*}\, dt$$

$$\langle x, A^* y \rangle_{L^2(\mathbb{R})} = \int_{-\infty}^{+\infty} x(t) \left((A^* y)(t) \right)^* dt$$

$$= \sum_{n\in\mathbb{Z}} \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} x(t) \underbrace{\left((A^* y)(t) \right)^*} dt$$

$$(A^* y)(t) = y_n\ ,\ t \in \left[n-\tfrac{1}{2}, n+\tfrac{1}{2}\right]$$

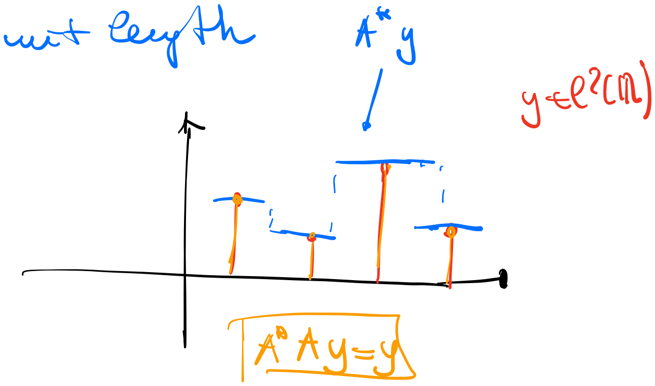
take the value of y_n and it creates step-functions of interval of length 1 for $t \in \left[n-\frac{1}{2}, n+\frac{1}{2}\right]$ (non-overlapping intervals)

↳ VEDERE CODICE

$A\,A^* : \ell^2(\mathbb{R}) \to \ell^2(\mathbb{Z})$ $\qquad$ $A^*A : L^2(\mathbb{R}) \to L^2(\mathbb{R})$

continuance operator over unit length

local averaging + sampling



$A : L^2(\mathbb{R}) \to \ell^2(\mathbb{Z})$

$A^* : \ell^2(\mathbb{Z}) \to L^2(\mathbb{R})$

$A^*A = id \Rightarrow$ (I as matrix)

AA^* orthogonal operator

BEST LEAST SQUARE APPROXIMATION USING PIECEWISE CONSTANT FUNCTIONS OVER UNIT INTERVAL.