PROJECTIONS

$A: \mathbb{R}^3 \to \mathbb{R}^2$ $\qquad\qquad$ $B: \mathbb{R}^2 \to \mathbb{R}^3$

$$A = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

If A left inverse of B $\Rightarrow$ $AB = I$ $\qquad$ $AB: \mathbb{R}^2 \to \mathbb{R}^2$

then by thm. $P = BA$ projection operator.

$$AB = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = I$$

$$P = BA = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$BA: \mathbb{R}^3 \to \mathbb{R}^3$

$$P^2 = \left(\frac{1}{2}\right)^2 \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & 2 & -2 \\ 0 & 4 & 0 \\ -2 & 2 & 2 \end{bmatrix}$$

idempotent

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} = P$$

$$P^* = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \neq P$$

$\Rightarrow$ NOT ORTHOGONAL

OBLIQUE

let us take $x \in \mathbb{R}^3$, $x = (x_0, x_1, x_2)$ we want to compute $R(P)$ (range of $P \Rightarrow$ the subspace that generates)

$$Px = \frac{1}{2}\begin{bmatrix}1 & 1 & -1\\ 0 & 2 & 0\\ -1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_0\\ x_1\\ x_2\end{bmatrix} = \begin{bmatrix}\overbrace{\tfrac{1}{2}(x_0+x_1-x_2)}^{\hat{x}_0}\\ \tfrac{1}{2}(2x_1)\ \hat{x}_1\\ \underbrace{\tfrac{1}{2}(-x_0+x_1+x_2)}_{\hat{x}_2}\end{bmatrix}$$

(1)+(3)

$$\begin{cases}\hat{x}_0+\hat{x}_2 = \frac{1}{2}(\cancel{x_0}+x_1\cancel{-x_2}) + \frac{1}{2}(\cancel{-x_0}+x_1\cancel{+x_2}) = x_1 = \hat{x}_1\\ \hat{x}_1 = x_1\end{cases}$$

eq. plane

Eq.: $\hat{x}_0+\hat{x}_2 = \hat{x}_1 \iff x_0+x_2 = x_1 \iff \boxed{x_0-x_1+x_2=0}$

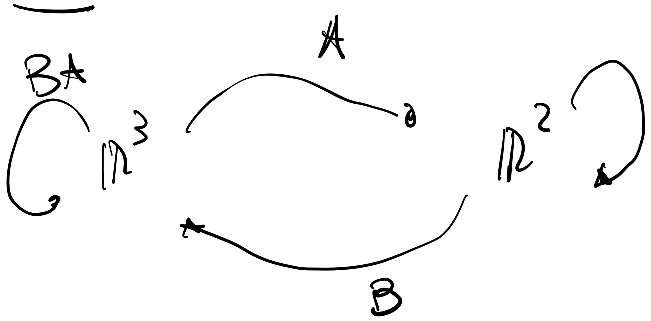
Es.

let $B = \begin{bmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix}$

Find all projection operators onto range of B; specify the orthogonal one.

$B: \mathbb{R}^2 \to \mathbb{R}^3$

$A: \mathbb{R}^3 \to \mathbb{R}^2$



$$\begin{cases}ABy = y\\ BAx = \hat{x}\\ BA\hat{x} = \hat{x}\end{cases}$$

$\hat{x} \in R(B) \subset \mathbb{R}^3$

Find all A's that are left inverse of $B \Rightarrow AB = I$

If $(BA)^* = BA \Rightarrow$ BA orthogonal

$$\begin{bmatrix} e_{00} & e_{01} & e_{02} \\ e_{10} & e_{11} & e_{12} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\begin{cases} e_{00} + e_{01} = 1 \\ e_{01} + e_{02} = 0 \\ e_{10} + e_{11} = 0 \\ e_{11} + e_{12} = 1 \end{cases}$$

$e_{01} = \alpha \qquad e_{11} = \beta$

$$\longrightarrow \begin{cases} e_{00} = 1 - \alpha \\ e_{02} = -\alpha \\ e_{10} = -\beta \\ e_{12} = 1 - \beta \end{cases} \qquad \alpha, \beta \in \mathbb{R}$$

$$\overset{B}{\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1-\alpha & \alpha & -\alpha \\ -\beta & \beta & 1-\beta \end{bmatrix}}$$

$$= \begin{bmatrix} 1-\alpha & \alpha & -\alpha \\ 1-\alpha-\beta & \alpha+\beta & 1-\alpha-\beta \\ -\beta & \beta & 1-\beta \end{bmatrix}$$

projection operators $\quad \infty^2$

$(BA)^* = BA$ (orthogonal)

$\alpha = \beta \; , \quad 1 - 2\alpha = \alpha \Rightarrow \alpha = \beta = \frac{1}{3}$

$$A = \frac{1}{3} \begin{bmatrix} 2 & 1 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$BA = \frac{1}{3} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$