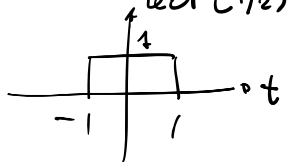
$$g(t) = \mathrm{rect}(t/2)$$



$T_s = 1$

tempo di simbolo

$$x(t) = \sum_k a_k \, g(t - kT_s)$$

$$(a_0, a_1, \dots, a_8) = (\overset{0}{-1}, \overset{1}{+1}, \overset{2}{+1}, \overset{3}{-1}, \overset{4}{-1}, \overset{5}{-1}, \overset{6}{+1}, \overset{7}{-1}, \overset{8}{+1})$$

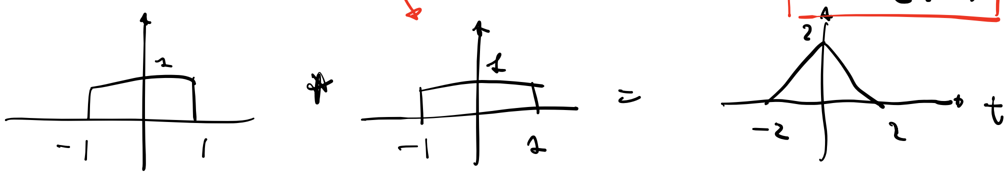
Nyquist

Filtro di ricezione

$$f(t) = g(t) * h(t) = g(t) * \underbrace{g^*(-t)}$$

$g^*(-t) = g(t) = \mathrm{rect}(t/2)$

$$= \mathrm{rect}(t/2) * \mathrm{rect}(t/2)$$



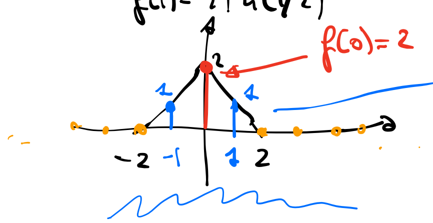
$\boxed{2\,\mathrm{tri}(t/2)}$

$$f(t) = 2\,\mathrm{tri}(t/2)$$

Condizione Nyquist ISI nulla

$$f(0) \neq 0 \qquad f(n\overset{1}{T_s}) = 0 \quad \forall n \neq 0, \; n \in \mathbb{Z}$$

$f(t) = 2\,\mathrm{tri}(t/2)$



$f(0) = 2$

$f(\pm 1) = 1$

$y(6) = ?$

in uscita di ricezione

$$y(t) = \sum_k a_k f(t - k\overbrace{T_s}^{=1})$$

$$= \sum_k a_k f(t-k)$$

$$= \sum_k a_k \left[2\, tri\left(\frac{t-k}{2}\right)\right]$$

$$= 2 \sum_k a_k \, tri\left(\frac{t-k}{2}\right)$$

$$y(nT_s) = y(n) = 2 \sum_k a_k \, \underbrace{tri\left(\frac{n-k}{2}\right)}$$

$n \in \mathbb{Z}$

$$tri\left(\frac{n-k}{2}\right) = \begin{cases} 1 & k = n \\ \frac{1}{2} & n-k = \pm 1 \rightarrow k = n \pm 1 \\ 0 & \text{altrimenti} \end{cases}$$

$k = n+1$

$$tri\left(\frac{n-(n+1)}{2}\right) = tri\left(-\frac{1}{2}\right) = \frac{1}{2}$$

$$= 2\left[a_{n-1} \overbrace{tri\left(\underbrace{\frac{n-(n-1)}{2}}_{\frac{1}{2}}\right)}^{\frac{1}{2}} + a_n \overbrace{tri\left(\underbrace{\frac{n-n}{2}}_{0}\right)}^{=1} + a_{n+1} \overbrace{tri\left(\underbrace{\frac{n-(n+1)}{2}}_{-\frac{1}{2}}\right)}^{\frac{1}{2}} \right]$$

$$= 2\left[a_{n-1} \cdot \frac{1}{2} + a_n + a_{n+1} \cdot \frac{1}{2} \right] = \boxed{a_{n-1} + 2a_n + a_{n+1}}$$

$$y(6) = a_5 + 2a_6 + a_7 = -1 + 2 \cdot 1 - 1 = \boxed{0}$$

4-PAM con codifica di Gray

$$x(t) = \sum_n a_n\, g(t - nT_s)$$

Assumiamo che $T_s = 1\,\mu s$, $E_g = 1$, $N_0 = \frac{1}{10}$

W_g energia dell'impulso base $g(t)$

Costellazione 4-PAM con livelli

$$a_k \in \{-3E_g, -E_g, +E_g, +3E_g\}$$

$$a_k \in \{-3, -1, +1, +3\}$$

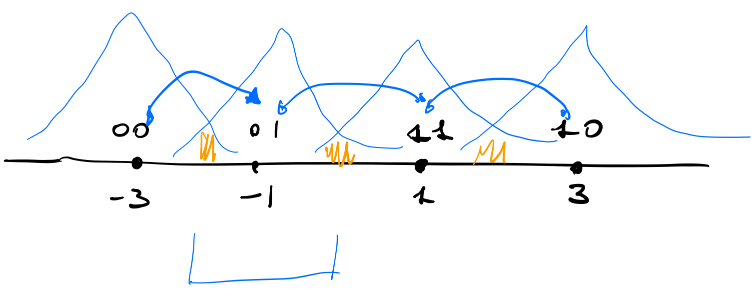
Livello	bit
−3	00
−1	01
+1	11
+3	10

codifica di Gray

prob. avere errore sul bit

$$P_b = \frac{P_s}{2}$$

nel simbolo



Codifica di Gray

1 bit

0
1

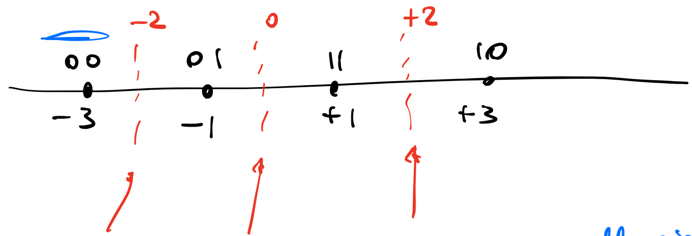
2 bit → 4 simboli

0	0
1	0
1	1
0	1

3 bit → 8 simboli = 3 bit

0	0	0
1	0	0
1	1	0
0	1	0
0	1	1
1	1	1
1	0	1
0	0	1

a) Disegnare costellazione 4-PAM all'uscita del filtro e indicare le soglie di decisione:



soglie ottime

coeff. ricostruito

Intervallo di decisione	$\hat{a}_n$	bit
$y(n) < -2$	-3	00
$-2 \leq y(n) < 0$	-1	01
$0 \leq y(n) < 2$	$+1$	11
$y(n) \geq 2$	$+3$	10

b) Probabilità di errore condizionata a ciascun livello

Varianza del rumore in uscita del filtro adatto

$$\sigma^2 = \frac{N_0}{2} E_g = \frac{N_0}{2} = \frac{1/10}{2} = \boxed{\frac{1}{20}}$$

in testa

$$P_{s=-3} = Q\left(\frac{E_g}{\sqrt{\frac{N_0}{2}E_g}}\right) = Q\left(\frac{1}{\sqrt{\frac{1}{20}}}\right) = Q\left(\sqrt{20}\right)$$

prob. errore se invio il simbolo $\boxed{-3}$

$P_{s=3} = Q(\sqrt{20})$ $\qquad$ $P_{s=\pm 1} = 2\,Q(\sqrt{20})$

in coda

c) Probabilità media di errore di simbolo P_s

$$P_s = \frac{1}{4}\left[P_{s=-3} + P_{s=-1} + P_{s=+1} + P_{s=+3} \right]$$

$$= \frac{1}{4}\left[\underbrace{Q(\sqrt{20}) + 2Q(\sqrt{20}) + 2Q(\sqrt{20}) + Q(\sqrt{20})}_{2(M-1)\,Q(\sqrt{20})} \right] \quad M=4$$

$$= \frac{2(M-1)}{M}\, Q\left(\frac{E_g}{\sqrt{\frac{N_0}{2} E_g}}\right)$$

$$= \frac{1}{\cancel{4}_2} \cdot \cancel{6}^{3} \cdot Q(\sqrt{20}) = \frac{3}{2}\, Q(\sqrt{20})$$

d) Calcolare P_b

Ogni simbolo trasporta 2 bit e simboli adiacenti differiscono in esattamente 1 bit

M-PAM

$M = 2^b$

$$P_b = \frac{P_s}{2} = \frac{3}{4}\, Q(\sqrt{20})$$

$$P_b = \frac{P_s}{\log_2 M}$$

e) Energia media per simbolo E_s e per bit E_b

R_s (ritmo di simbolo) e R_b (bit-rate)

$$E_{a_k} = \boxed{a_k^2 \, E_g} \Rightarrow a_k^2$$

simbolo a_k

$$E_s = \frac{1}{4}\left[E_{-3} + E_{-1} + E_{+1} + E_{+3} \right]$$

$$= \frac{1}{4}\left[(-3)^2 + (-1)^2 + (1)^2 + (3)^2 \right] = \frac{1}{4}\left[9 + 1 + 1 + 9 \right]$$

$$= \frac{20}{4} = \boxed{5}$$

$$E_b = \frac{5}{2}$$

#bit per simbolo

$$R_s = \frac{1}{T_s} = \frac{1}{1\mu s} = 1 \cdot 10^6 \; sym/s = \underline{1 \, M \, sym/s}$$

tempo di simbolo

$$R_b = 2 \, R_s = 2 \; Mbit/s$$

Es. Scelta Modulazione M-PAM

$R_b = 1.5$ Mbit/s, canale AWGN (additivo gaussiano bianco) con banda disponibile $B_c = 330$ kHz.

\# bit per simb.

Si consideri una trasmissione M-PAM con $M = 2^b$

Si assume inizialmente impulso ideale di Nyquist (roll-off nullo).

a) Verificare se 2-PAM, 4-PAM e 8-PAM sono compatibili con la banda B_c.

impulso ideale

$$B_{min} = \boxed{\frac{R_s}{2}} = \frac{R_b}{2\log_2 M}$$

$R_b = 1.5$ Mbit/s

$$\boxed{R_s = \frac{R_b}{\log_2 M}}$$

	R_s	$B_{min} = R_s/2$
2-PAM	$1.5/1 = 1.5 \cdot 10^6$ Hz	$1.5/2 = 750$ kHz
4-PAM	$1.5/2$	$1.5/4 = 375$ kHz
8-PAM	$1.5/3$	$1.5/6 = \boxed{250}$ kHz

$B_c = 330$ kHz

8-PAM è compatibile

Impulso a coseno rialzato con roll-off α

8-PAM

$$\underbrace{B_{min,\alpha}}_{\text{con coseno rialzato } \alpha \text{ (roll-off)}} = (1+\alpha)\boxed{B_{min}}$$

↑ con impulso ideale

$$= (1+\alpha)\, 250$$

Qual è il roll-off massimo che posso avere?

$$(1+\alpha)250 < 330$$

$$250\alpha < \underbrace{330-250}_{80}$$

$$\alpha < \frac{80}{250} \sim \boxed{0.32}$$

Es. Codifica di Sorgente + Hamming + 4-QAM

Si consideri una sorgente discreta senza memoria

$$\mathcal{A} = \{A, B, C, D, E\}$$

$$p_A = \frac{1}{2} \qquad p_B = p_C = p_D = p_E = \frac{1}{8}$$

prob. simboli

Si consideri la sequenza di 8 simboli

ACBBADEE

1) Huffman per la sorgente

2) Codice Hamming $(\overset{N}{7}, \overset{K}{4})$ sistematico, suddividere in blocchi di 4bit la sequenza e applicare il mapping:

$$\underbrace{(u_1, u_2, u_3, u_4)}_{\text{4 bit informativi}} \longrightarrow (u_1, u_2, u_3, u_4, \underbrace{p_1, p_2, p_3}_{\text{bit parità}})$$

$$p_1 = u_1 \oplus u_2 \oplus u_3$$

$$p_2 = u_2 \oplus u_3 \oplus u_4$$

$$p_3 = u_1 \oplus u_2 \oplus u_4$$

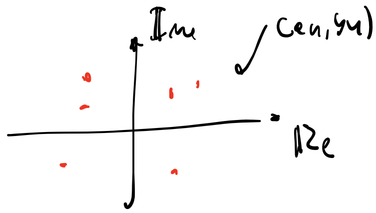
3) Modulazione 4-QAM

coeff. fase

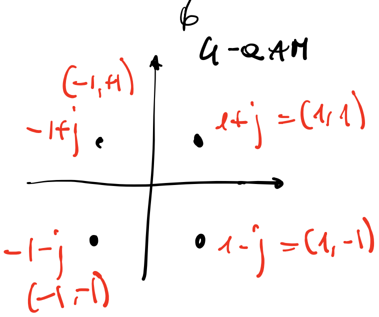
$$x(t) = \sum_n \boxed{a_n}\, p(t - nT_s) \cos(2\pi f_0 t) + \sum_n \boxed{b_n}\, p(t - nT_s) \sin(2\pi f_0 t)$$

Bit	a_n	b_n
00	+1	+1
01	+1	−1
11	−1	−1
10	−1	+1

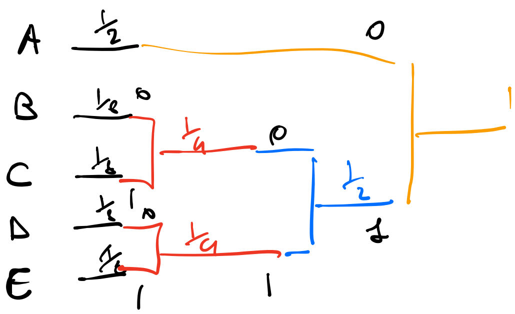
coeff. quadratura



$a_n + j\,b_n$



a) Huffman



	Bit
A	0
B	100
C	101
D	110
E	111

A C BB A D

↓ codificate

$\underbrace{0}_{A}\ \underbrace{101}_{C}\ \underbrace{100}_{B}\ \underbrace{100}_{B}\ \underbrace{0}_{A}\ \underbrace{110}_{D}\ \underbrace{111}_{E}\ \underbrace{111}_{E}$

0101 1001 0001 1011 1111

$\underbrace{0101}_{4}$ $\underbrace{1001}_{4}$ $\underbrace{0001}_{4}$ $\underbrace{1011}_{4}$ $\underbrace{1111}_{4}$

0101100 1001110 0001011 - - -

Hamming

Blocco informativo	PARITÀ (p1,p2,p3)	Parole
0 1 0 1	1 0 0	0101 ; 100
1 0 0 1	1 1 0	1001 ; 110
0 0 0 1	0 1 1	0001 ; 011
1 0 1 1	0 0 0	1011 ; 000
1 1 1 1	1 1 1	1111 ; 111

Primi 8 bit rep. Hamming

01 01 10 01

QAM

$\underbrace{1-j \quad 1-j \quad -1+j \quad 1-j}$ → simboli QAM